US009832085B1

(12) **United States Patent**
Malboubi

(10) **Patent No.: US 9,832,085 B1**
(45) **Date of Patent: Nov. 28, 2017**

(54) **SYSTEM FOR ESTIMATING UNKNOWN ATTRIBUTES OF INTEREST IN THE UNDER-DETERMINED INVERSE PROBLEM AND A PROCESS OF ACCOMPLISHING THE SAME**

(71) Applicant: **Mehdi Malboubi**, San Ramon, CA (US)

(72) Inventor: **Mehdi Malboubi**, San Ramon, CA (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 129 days.

(21) Appl. No.: **15/002,376**

(22) Filed: **Jan. 20, 2016**

Related U.S. Application Data

(60) Provisional application No. 62/105,437, filed on Jan. 20, 2015.

(51) **Int. Cl.**
H04L 12/26 (2006.01)

(52) **U.S. Cl.**
CPC ***H04L 43/045*** (2013.01); ***H04L 43/0829*** (2013.01); ***H04L 43/0852*** (2013.01); ***H04L 43/0888*** (2013.01)

(58) **Field of Classification Search**
CPC H04L 45/38
USPC 370/252
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

6,785,240 B1 * 8/2004 Cao H04L 43/045 370/238
7,746,784 B2 * 6/2010 de Heer G06F 9/505 370/229
7,836,201 B2 * 11/2010 Kotrla H04L 41/0896 709/203
8,125,911 B2 * 2/2012 Patel H04L 12/5691 370/241
8,248,925 B2 * 8/2012 Allan H04L 45/00 370/229

(Continued)

OTHER PUBLICATIONS

M. Malboubi. L. Wang, C. Chuah, and P. Sharma, "Intelligent sdn based traffic (de)aggregation and measurement paradigm (istamp)," IEEE INFOCOM, 2014.

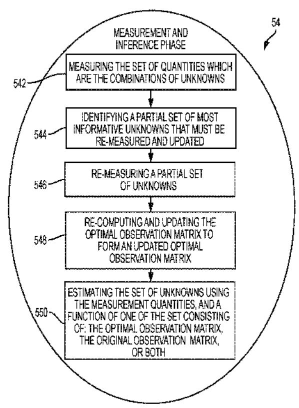
(Continued)

Primary Examiner — Guang Li
(74) *Attorney, Agent, or Firm* — Law Office of Michael O'Brien

(57) **ABSTRACT**

A method for solving an under-determined inverse problem or network inference/tomography problem in per-flow size, delay, loss and throughput inference in a computer network, through a system is presented. The method includes the following steps, which are not necessarily in order. First, establishing the computer network having a plurality of nodes wherein the per-flow size, the delay, the loss and the throughput inference are unknown. An original observation or routing matrix determines how flows are appeared on the links and construct the measurements. Next, performing a learning phase to obtain an optimal observation matrix or pseudo-optimal observation matrix. After that, performing a computer controller adaptive measurement and inference phase to estimate the set of unknowns using the measurement quantities, and a function of one of the set consisting of: the optimal observation matrix, the original observation matrix, or both.

15 Claims, 17 Drawing Sheets



(56) **References Cited**

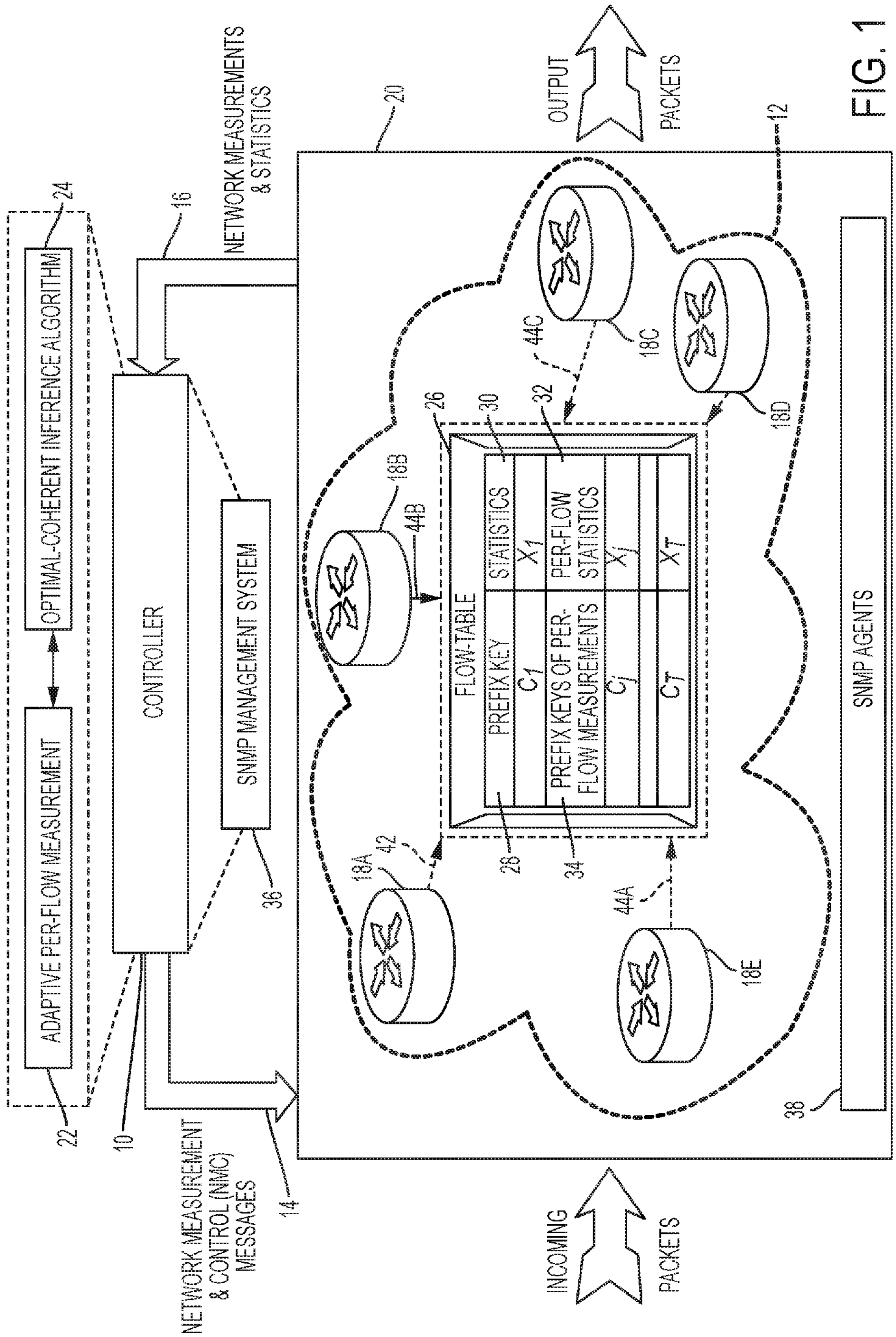
U.S. PATENT DOCUMENTS

8,711,676	B2 *	4/2014	Bhat	H04L 41/0668 370/216
9,083,627	B2 *	7/2015	Vasseur	H04L 45/125
9,641,452	B2 *	5/2017	Lu	H04L 47/788
9,646,253	B2 *	5/2017	Kankar	G06N 5/048
2007/0177506	A1 *	8/2007	Singer	H04L 45/38 370/235
2012/0127029	A1 *	5/2012	Rachlin	G01S 19/40 342/357.23
2012/0191843	A1 *	7/2012	Ding	G06F 11/3006 709/224
2013/0262661	A1 *	10/2013	Malboubi	H04L 41/142 709/224

OTHER PUBLICATIONS

M. Moshref, M. Yu, R. Govindan, and A. Vandat, "Dream: Dynamic resource allocation for software-defined measurement," ACM SIGCOMM (HozSDN), 2014.
A. Soule, K. Salamatian, and N. Taft, "Combining filtering and statistical methods for anomaly detection," ACM SIGCOMM, 2005.
M. Roughan, M. Thorup, and Y. Zhang, "Traffic engineering with estimated traffic matrices," ACM-IMC, 2003.
"Cisco netflow," At: http://en.wikipedia.org/wiki/Netflow.
"Sampled netflow," At: http://www.cisco.com/en/US/docs/ios/12\0s/feature/guide/12s\sanf.html.
A. Kumar and J. Xu, "Sketch guided sampling—using on-line estimates of flow size for adaptive data collection," in INFOCOM, 2006.
M. Roughan, "A case study of the accuracy of snmp measurements," JECE, vol. 2010, pp. 33:1-33:7, Jan. 2010.
A. Medina and et al., "Traffic matrix estimation:existing techniques and new directions," in ACM SIGCOMM, 2002.
R. M, Y. Zhang, W. Willinger, and L. Qiu, "Spatio-temporal compressive sensing and internet traffic matrices," IEEE/ACM Transactions on Networking, vol. 20, pp. 662-676, 2012.
Q. Zhao, Z. Ge, J. Wang, and J. Xu, "Robust traffic matrix estimation with imperfect information: Making use of multiple data sources," ACM-SIGMETRICS, 2006.
J. Cao, D. Davis, S. Wiel, and B. Yu, "Time varying network tomography: Router link data," JNL of American Stat. Association, 2000.
G. Huang, A. Lall, C.-N. Chuah, and J. Xu, "Uncovering global icebergs in distributed streams: Results and implications," J. Network Syst. Manage, vol. 19, No. 1, pp. 84-110, 2011.
A. Tootoonchian, M. Ghobadi, and Y. Ganjali, "Opentm: traffic matrix estimator for openfiow networks," 2010.
N. van Adrichen, C. Doerr, and F. Kuipers, "OpenNetMon: Network monitoring in openflow sdn networks," IEEE, INFOCOM, 2014.
M. Yu, L. Jose, and R. Miao. "Software defined traffic measurement with opensketch," ACM-USENIX, 2013.
L. Jose. M. Yu, and J. Rexford, "Online measurement of large traffic aggregates on commodity switches," ACM-Hot-ICE, 2011.
M. C. A.P. Lakhina. E. K. C.M. Diot, and N. Taft, "Structural analysis of network traffic flows," ACM Sigmetrics, 2004.
A. Israel and T. E Greville in Generalized Inverses: Theory and Applications, Springer, 2003.
H. Nguyen and P. Thiran, "Network loss inference with second order statistics of end-to-end flows," ACM-IMC, 2007.
M. Basseville and I. Nikiforov in Detection of Abrupt Changes: Theory and Application, Prentice-Hall, 1993.
"Geant network:." http://totem.info.ucl.ac.be/dataset.html.
"Miniet," At: http://mininet.org/.

* cited by examiner



22
ADAPTIVE PER-FLOW MEASUREMENT
OPTIMAL-COHERENT INFERENCE ALGORITHM
24
10
CONTROLLER
SNMP MANAGEMENT SYSTEM
36
16
NETWORK MEASUREMENTS & STATISTICS
NETWORK MEASUREMENT & CONTROL (NMC) MESSAGES
14
20
12
18A
18B
18C
18D
18E
42
44A
44B
44C
26
28
30
32
34
FLOW-TABLE
PREFIX KEY
STATISTICS
C_1
X_1
PREFIX KEYS OF PER-FLOW MEASUREMENTS
PER-FLOW STATISTICS
C_j
X_j
C_T
X_T
SNMP AGENTS
38
INCOMING PACKETS
OUTPUT PACKETS

FIG. 1

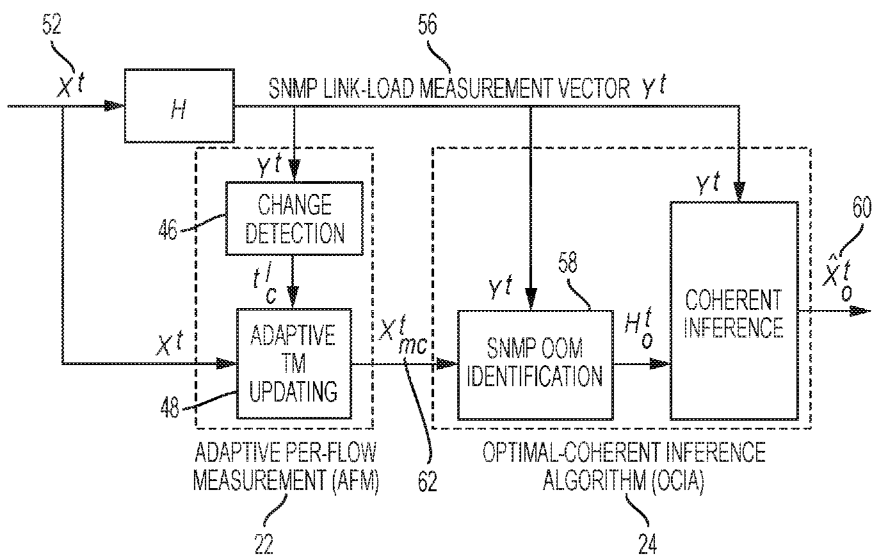

52
X^t
H
56
SNMP LINK-LOAD MEASUREMENT VECTOR Y^t
Y^t
CHANGE DETECTION
46
t_c^l
ADAPTIVE TM UPDATING
X^t
48
X^t_{mc}
62
ADAPTIVE PER-FLOW MEASUREMENT (AFM)
22
Y^t
58
SNMP OOM IDENTIFICATION
H_o^t
Y^t
COHERENT INFERENCE
60
$\hat{X}_o^t$
OPTIMAL-COHERENT INFERENCE ALGORITHM (OCIA)
24

FIG. 2

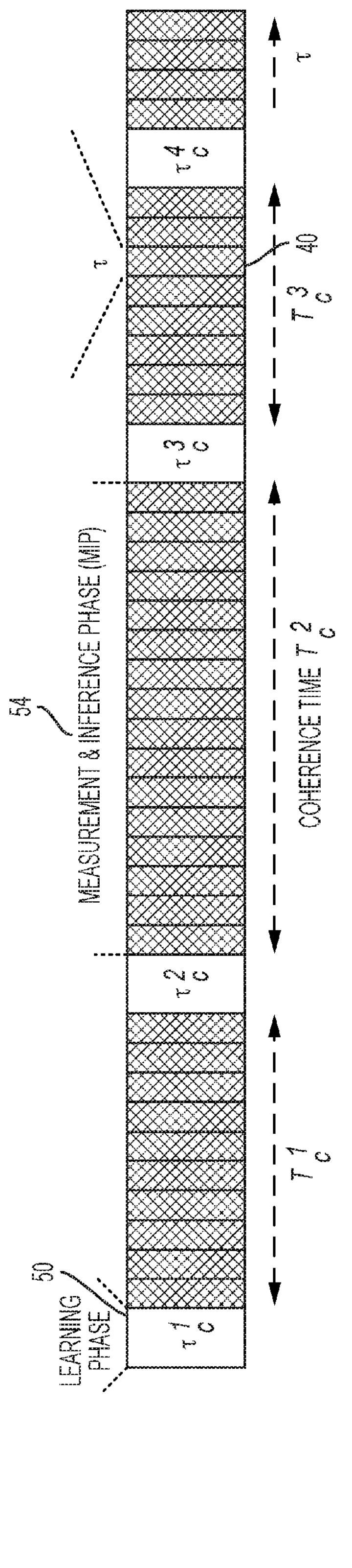

τ
τ c 4
τ
40
T c 3
τ c 3
MEASUREMENT & INFERENCE PHASE (MIP)
54
COHERENCE TIME T c 2
τ c 2
T c 1
50
LEARNING PHASE
τ c 1

FIG. 3

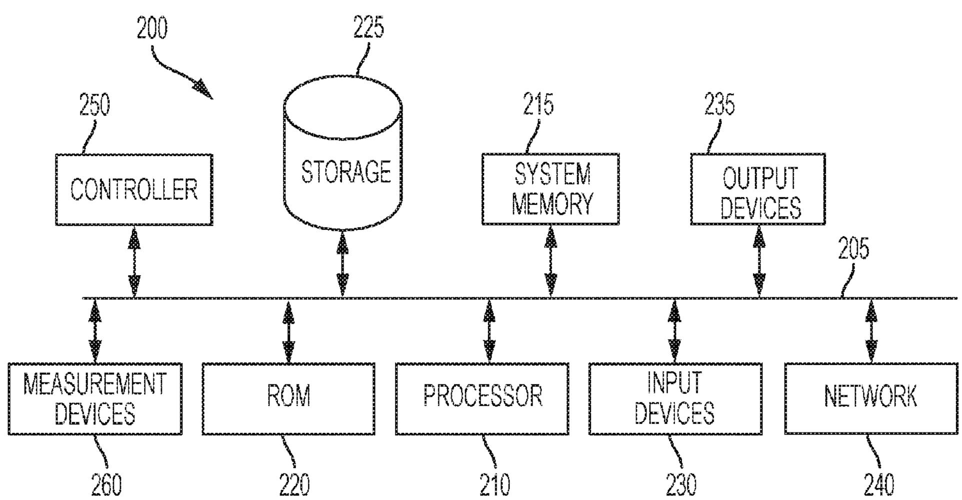

200
225
STORAGE
250
CONTROLLER
215
SYSTEM MEMORY
235
OUTPUT DEVICES
205
MEASUREMENT DEVICES
ROM
PROCESSOR
INPUT DEVICES
NETWORK
260
220
210
230
240

FIG. 4

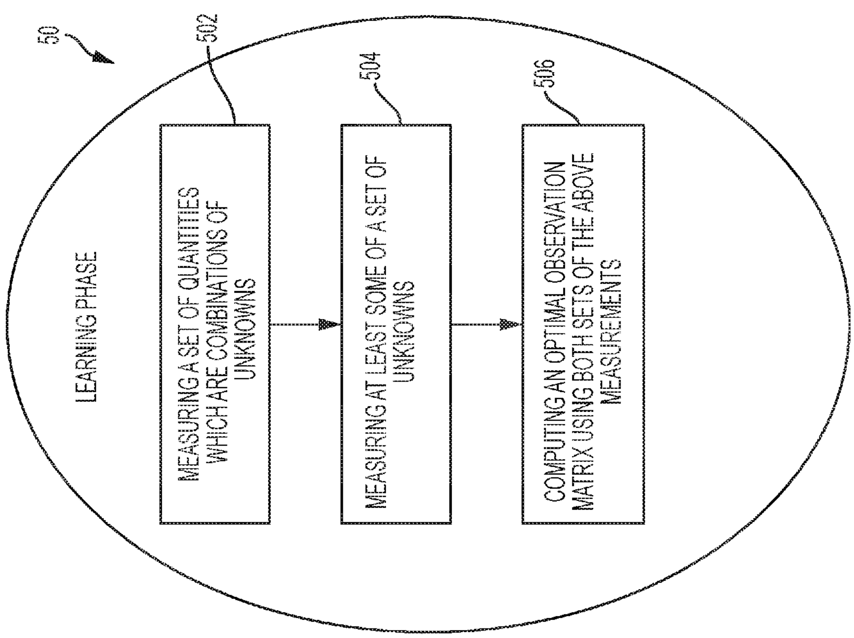

50
LEARNING PHASE
502
MEASURING A SET OF QUANTITIES WHICH ARE COMBINATIONS OF UNKNOWNS
504
MEASURING AT LEAST SOME OF A SET OF UNKNOWNS
506
COMPUTING AN OPTIMAL OBSERVATION MATRIX USING BOTH SETS OF THE ABOVE MEASUREMENTS

FIG. 5A

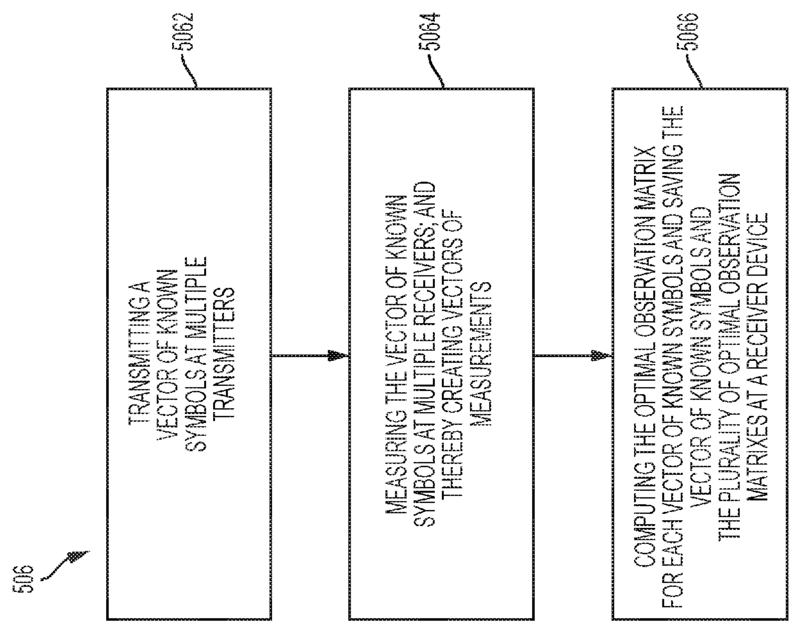

506
5062
TRANSMITTING A VECTOR OF KNOWN SYMBOLS AT MULTIPLE TRANSMITTERS
5064
MEASURING THE VECTOR OF KNOWN SYMBOLS AT MULTIPLE RECEIVERS; AND THEREBY CREATING VECTORS OF MEASUREMENTS
5066
COMPUTING THE OPTIMAL OBSERVATION MATRIX FOR EACH VECTOR OF KNOWN SYMBOLS AND SAVING THE VECTOR OF KNOWN SYMBOLS AND THE PLURALITY OF OPTIMAL OBSERVATION MATRIXES AT A RECEIVER DEVICE

FIG. 5B

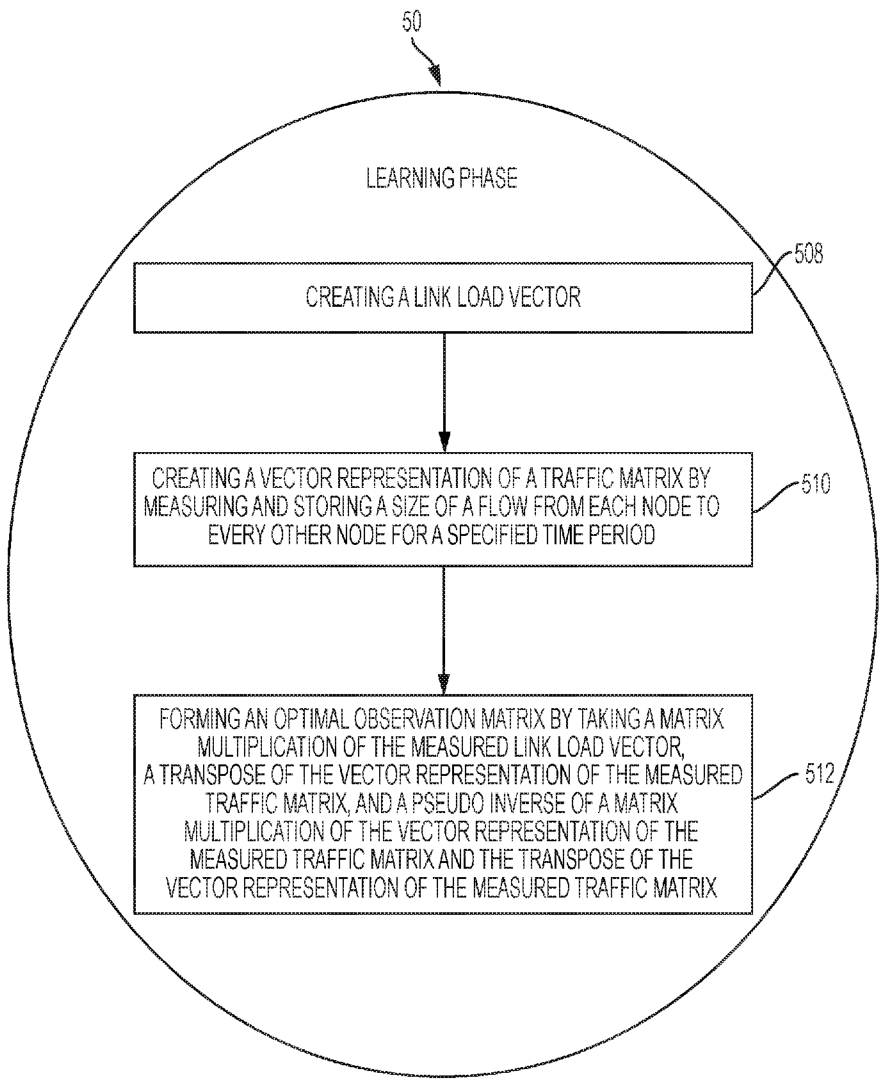

50
LEARNING PHASE
CREATING A LINK LOAD VECTOR
508
CREATING A VECTOR REPRESENTATION OF A TRAFFIC MATRIX BY MEASURING AND STORING A SIZE OF A FLOW FROM EACH NODE TO EVERY OTHER NODE FOR A SPECIFIED TIME PERIOD
510
FORMING AN OPTIMAL OBSERVATION MATRIX BY TAKING A MATRIX MULTIPLICATION OF THE MEASURED LINK LOAD VECTOR, A TRANSPOSE OF THE VECTOR REPRESENTATION OF THE MEASURED TRAFFIC MATRIX, AND A PSEUDO INVERSE OF A MATRIX MULTIPLICATION OF THE VECTOR REPRESENTATION OF THE MEASURED TRAFFIC MATRIX AND THE TRANSPOSE OF THE VECTOR REPRESENTATION OF THE MEASURED TRAFFIC MATRIX
512

FIG. 5C

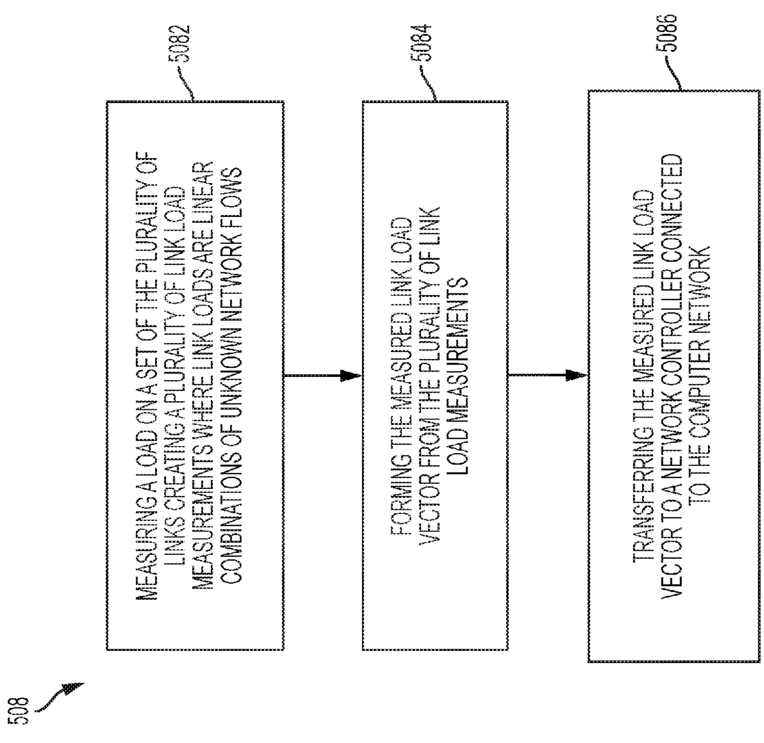

508
5082
MEASURING A LOAD ON A SET OF THE PLURALITY OF LINKS CREATING A PLURALITY OF LINK LOAD MEASUREMENTS WHERE LINK LOADS ARE LINEAR COMBINATIONS OF UNKNOWN NETWORK FLOWS
5084
FORMING THE MEASURED LINK LOAD VECTOR FROM THE PLURALITY OF LINK LOAD MEASUREMENTS
5086
TRANSFERRING THE MEASURED LINK LOAD VECTOR TO A NETWORK CONTROLLER CONNECTED TO THE COMPUTER NETWORK

FIG. 5D

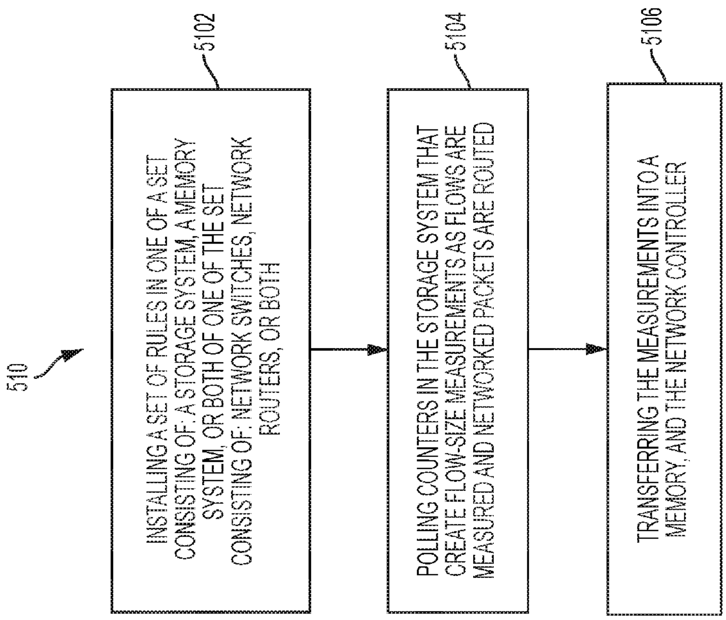

510
5102
INSTALLING A SET OF RULES IN ONE OF A SET CONSISTING OF: A STORAGE SYSTEM, A MEMORY SYSTEM, OR BOTH OF ONE OF THE SET CONSISTING OF: NETWORK SWITCHES, NETWORK ROUTERS, OR BOTH
5104
POLLING COUNTERS IN THE STORAGE SYSTEM THAT CREATE FLOW-SIZE MEASUREMENTS AS FLOWS ARE MEASURED AND NETWORKED PACKETS ARE ROUTED
5106
TRANSFERRING THE MEASUREMENTS INTO A MEMORY, AND THE NETWORK CONTROLLER

FIG. 5E

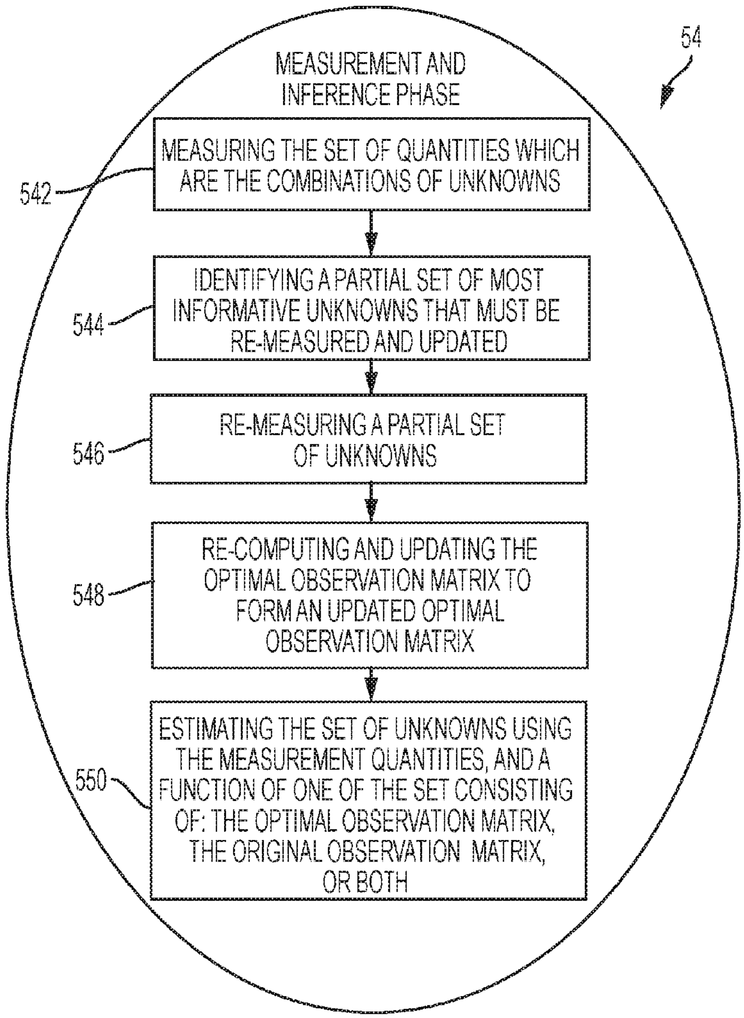

54
MEASUREMENT AND INFERENCE PHASE
542
MEASURING THE SET OF QUANTITIES WHICH ARE THE COMBINATIONS OF UNKNOWNS
544
IDENTIFYING A PARTIAL SET OF MOST INFORMATIVE UNKNOWNS THAT MUST BE RE-MEASURED AND UPDATED
546
RE-MEASURING A PARTIAL SET OF UNKNOWNS
548
RE-COMPUTING AND UPDATING THE OPTIMAL OBSERVATION MATRIX TO FORM AN UPDATED OPTIMAL OBSERVATION MATRIX
550
ESTIMATING THE SET OF UNKNOWNS USING THE MEASUREMENT QUANTITIES, AND A FUNCTION OF ONE OF THE SET CONSISTING OF: THE OPTIMAL OBSERVATION MATRIX, THE ORIGINAL OBSERVATION MATRIX, OR BOTH

FIG. 6A

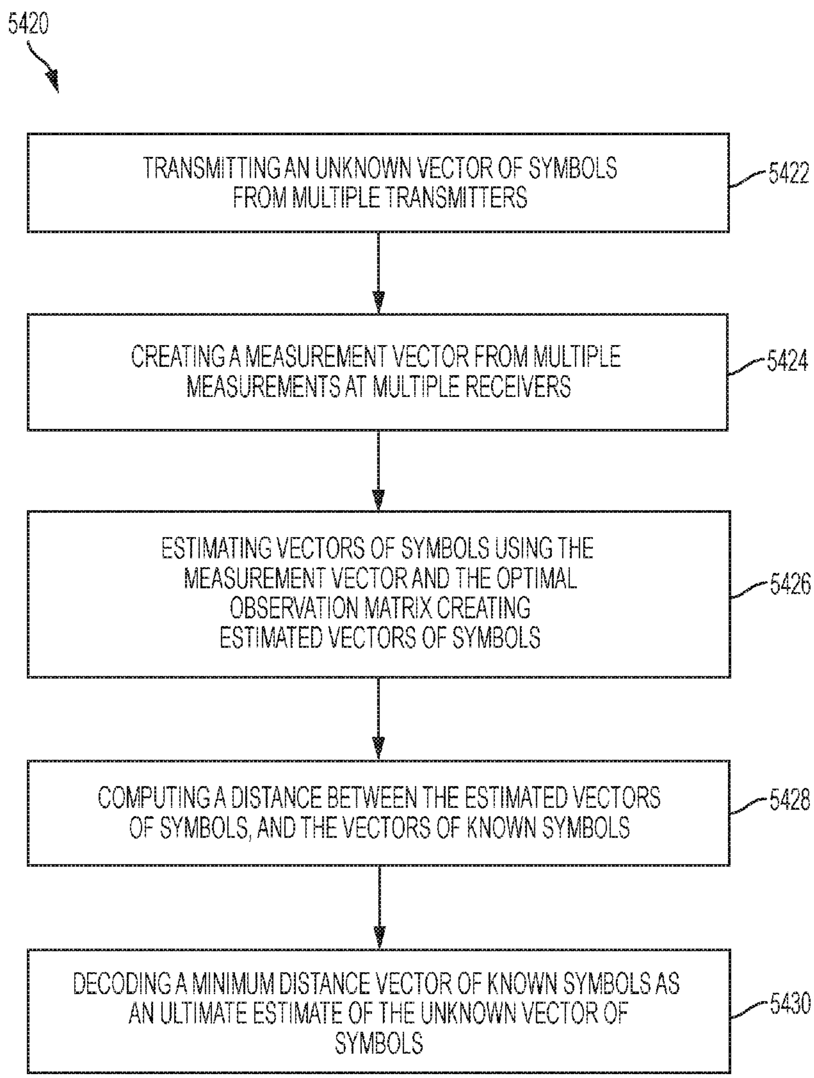

5420
TRANSMITTING AN UNKNOWN VECTOR OF SYMBOLS FROM MULTIPLE TRANSMITTERS
5422
CREATING A MEASUREMENT VECTOR FROM MULTIPLE MEASUREMENTS AT MULTIPLE RECEIVERS
5424
ESTIMATING VECTORS OF SYMBOLS USING THE MEASUREMENT VECTOR AND THE OPTIMAL OBSERVATION MATRIX CREATING ESTIMATED VECTORS OF SYMBOLS
5426
COMPUTING A DISTANCE BETWEEN THE ESTIMATED VECTORS OF SYMBOLS, AND THE VECTORS OF KNOWN SYMBOLS
5428
DECODING A MINIMUM DISTANCE VECTOR OF KNOWN SYMBOLS AS AN ULTIMATE ESTIMATE OF THE UNKNOWN VECTOR OF SYMBOLS
5430

FIG. 6B

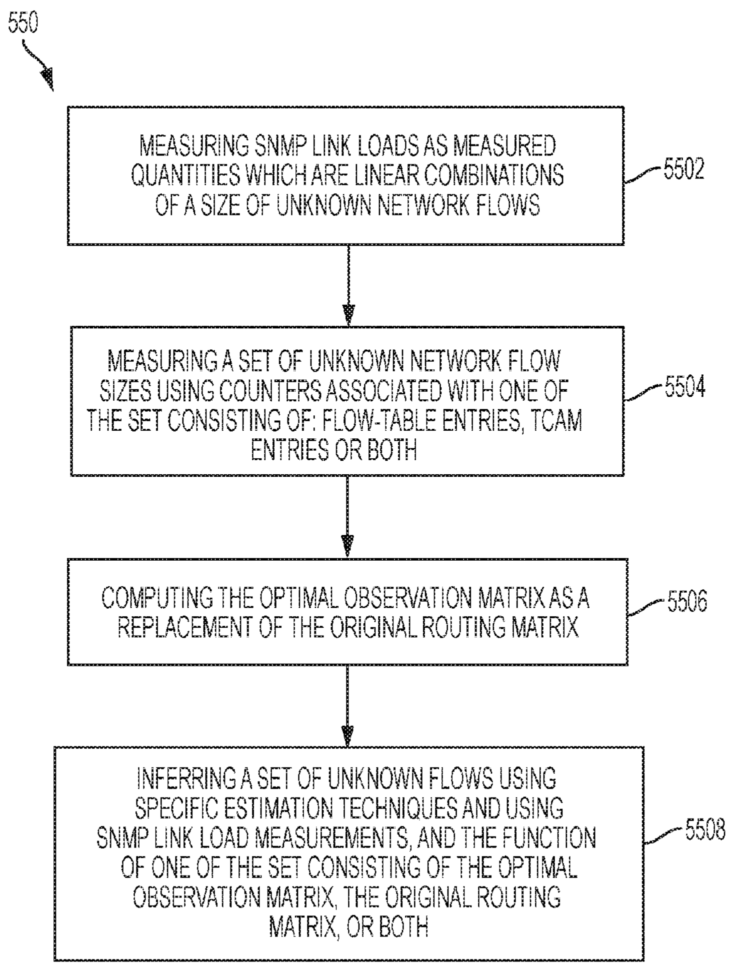

550
MEASURING SNMP LINK LOADS AS MEASURED QUANTITIES WHICH ARE LINEAR COMBINATIONS OF A SIZE OF UNKNOWN NETWORK FLOWS
5502
MEASURING A SET OF UNKNOWN NETWORK FLOW SIZES USING COUNTERS ASSOCIATED WITH ONE OF THE SET CONSISTING OF: FLOW-TABLE ENTRIES, TCAM ENTRIES OR BOTH
5504
COMPUTING THE OPTIMAL OBSERVATION MATRIX AS A REPLACEMENT OF THE ORIGINAL ROUTING MATRIX
5506
INFERRING A SET OF UNKNOWN FLOWS USING SPECIFIC ESTIMATION TECHNIQUES AND USING SNMP LINK LOAD MEASUREMENTS, AND THE FUNCTION OF ONE OF THE SET CONSISTING OF THE OPTIMAL OBSERVATION MATRIX, THE ORIGINAL ROUTING MATRIX, OR BOTH
5508

FIG. 6C

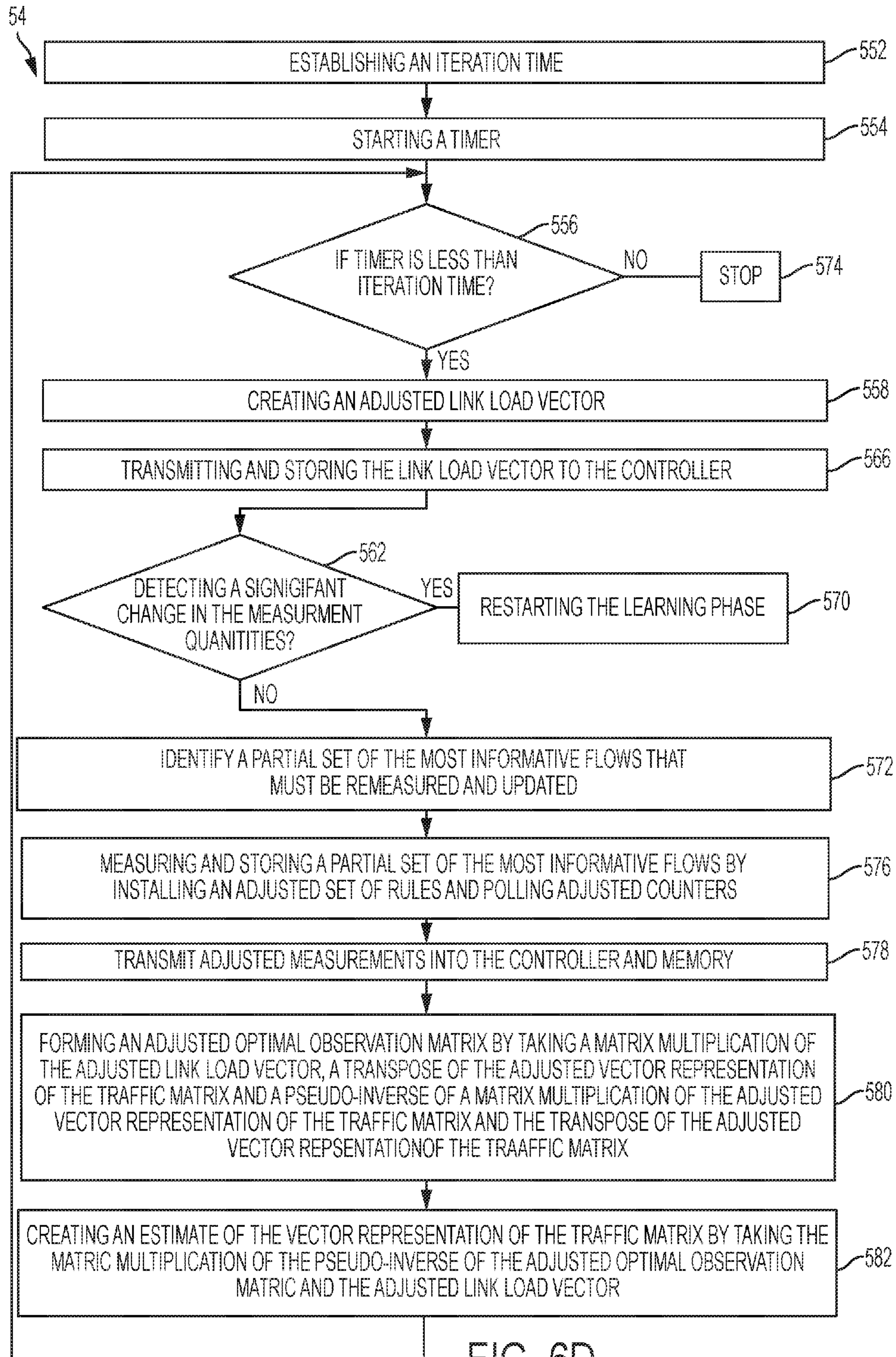

54
ESTABLISHING AN ITERATION TIME
552
STARTING A TIMER
554
IF TIMER IS LESS THAN ITERATION TIME?
556
NO
STOP
574
YES
CREATING AN ADJUSTED LINK LOAD VECTOR
558
TRANSMITTING AND STORING THE LINK LOAD VECTOR TO THE CONTROLLER
566
DETECTING A SIGNIGIFANT CHANGE IN THE MEASURMENT QUANITITIES?
562
YES
RESTARTING THE LEARNING PHASE
570
NO
IDENTIFY A PARTIAL SET OF THE MOST INFORMATIVE FLOWS THAT MUST BE REMEASURED AND UPDATED
572
MEASURING AND STORING A PARTIAL SET OF THE MOST INFORMATIVE FLOWS BY INSTALLING AN ADJUSTED SET OF RULES AND POLLING ADJUSTED COUNTERS
576
TRANSMIT ADJUSTED MEASUREMENTS INTO THE CONTROLLER AND MEMORY
578
FORMING AN ADJUSTED OPTIMAL OBSERVATION MATRIX BY TAKING A MATRIX MULTIPLICATION OF THE ADJUSTED LINK LOAD VECTOR, A TRANSPOSE OF THE ADJUSTED VECTOR REPRESENTATION OF THE TRAFFIC MATRIX AND A PSEUDO-INVERSE OF A MATRIX MULTIPLICATION OF THE ADJUSTED VECTOR REPRESENTATION OF THE TRAFFIC MATRIX AND THE TRANSPOSE OF THE ADJUSTED VECTOR REPSENTATIONOF THE TRAAFFIC MATRIX
580
CREATING AN ESTIMATE OF THE VECTOR REPRESENTATION OF THE TRAFFIC MATRIX BY TAKING THE MATRIC MULTIPLICATION OF THE PSEUDO-INVERSE OF THE ADJUSTED OPTIMAL OBSERVATION MATRIC AND THE ADJUSTED LINK LOAD VECTOR
582

FIG. 6D

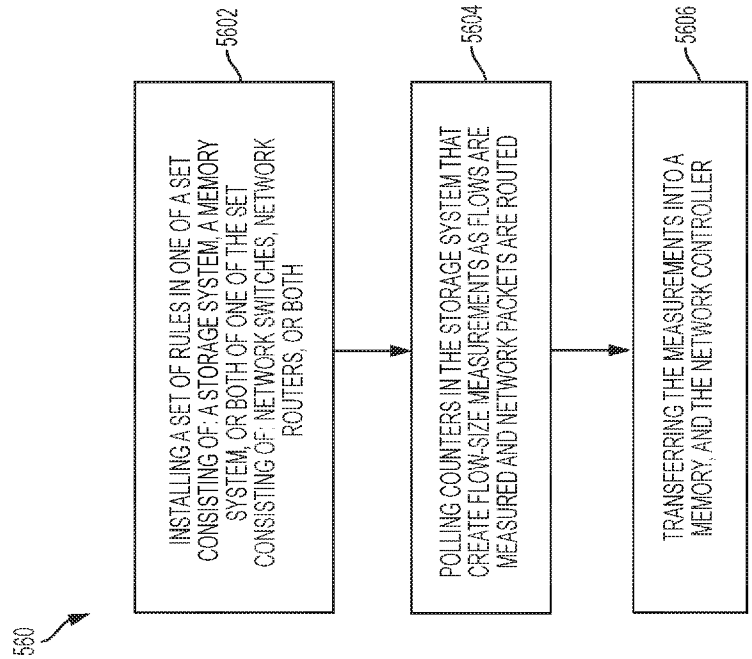

560
5602
INSTALLING A SET OF RULES IN ONE OF A SET CONSISTING OF: A STORAGE SYSTEM, A MEMORY SYSTEM, OR BOTH OF ONE OF THE SET CONSISTING OF: NETWORK SWITCHES, NETWORK ROUTERS, OR BOTH
5604
POLLING COUNTERS IN THE STORAGE SYSTEM THAT CREATE FLOW-SIZE MEASUREMENTS AS FLOWS ARE MEASURED AND NETWORK PACKETS ARE ROUTED
5606
TRANSFERRING THE MEASUREMENTS INTO A MEMORY, AND THE NETWORK CONTROLLER

FIG. 6F

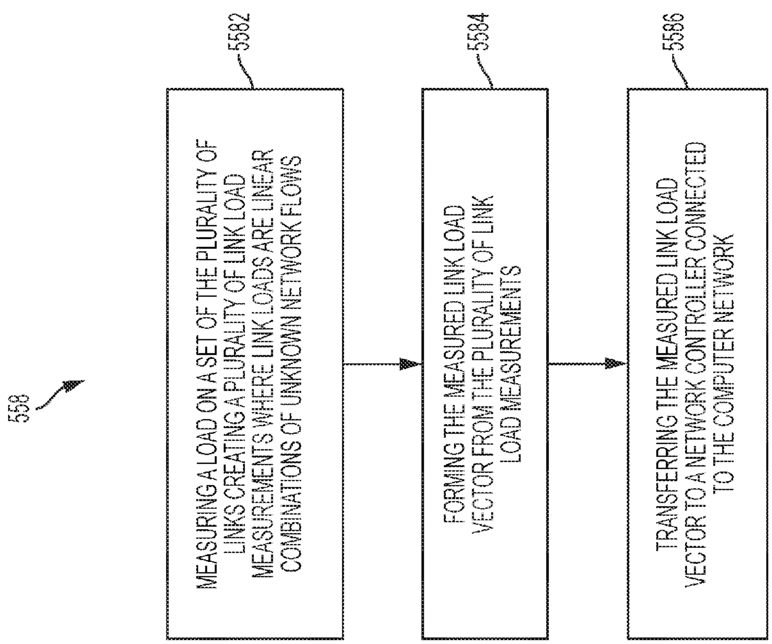

558
5582
MEASURING A LOAD ON A SET OF THE PLURALITY OF LINKS CREATING A PLURALITY OF LINK LOAD MEASUREMENTS WHERE LINK LOADS ARE LINEAR COMBINATIONS OF UNKNOWN NETWORK FLOWS
5584
FORMING THE MEASURED LINK LOAD VECTOR FROM THE PLURALITY OF LINK LOAD MEASUREMENTS
5586
TRANSFERRING THE MEASURED LINK LOAD VECTOR TO A NETWORK CONTROLLER CONNECTED TO THE COMPUTER NETWORK

FIG. 6E

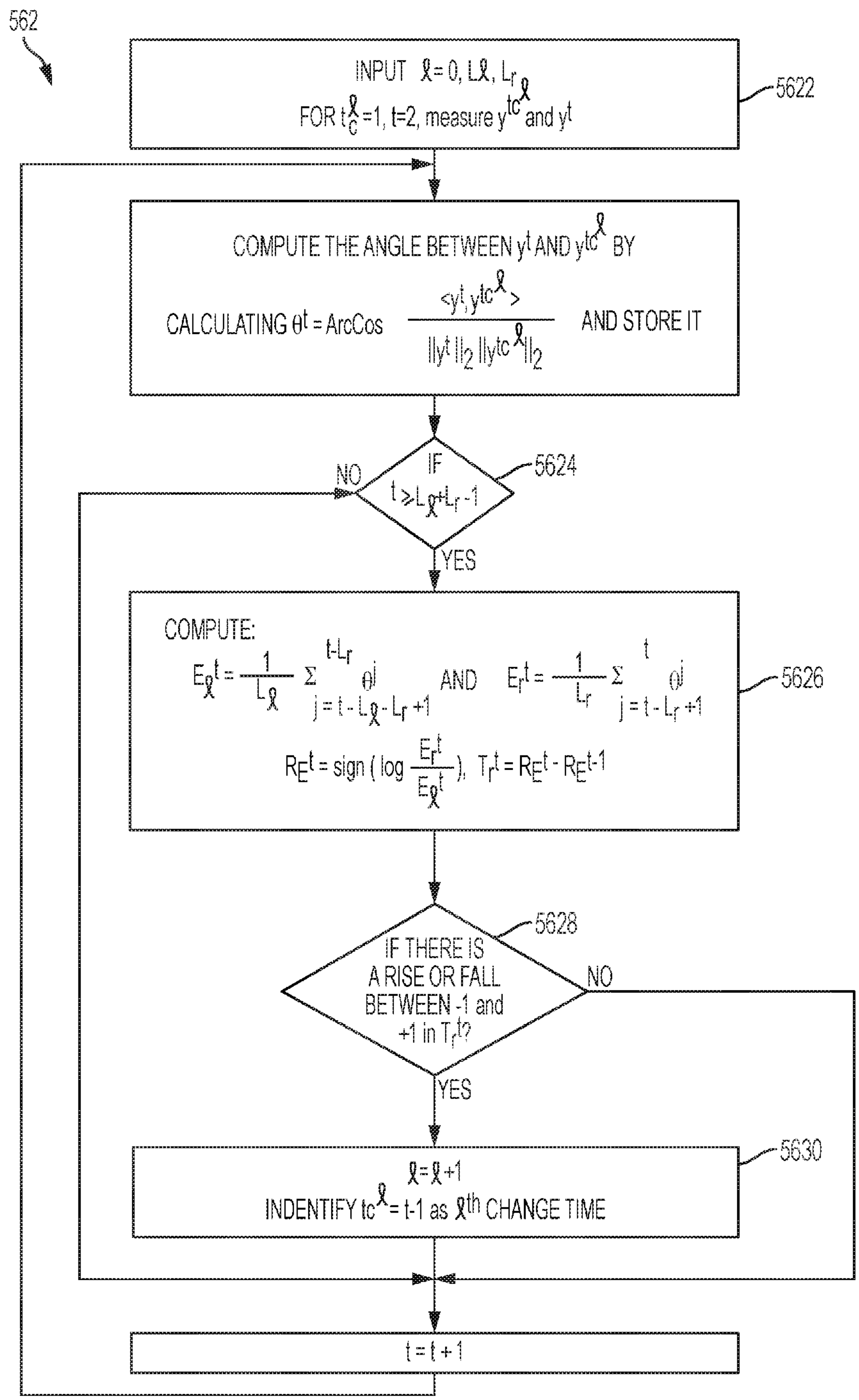

562
INPUT ℓ = 0, Lℓ, Lr
FOR tcℓ =1, t=2, measure ytcℓ and yt
5622
COMPUTE THE ANGLE BETWEEN yt AND ytcℓ BY
CALCULATING θt = ArcCos <yt, ytcℓ> / ||yt||2 ||ytcℓ||2 AND STORE IT
IF t ≥ Lℓ+Lr-1
5624
NO
YES
COMPUTE:
Eℓt = 1/Lℓ Σ θj (j = t - Lℓ - Lr + 1 to t-Lr) AND Ert = 1/Lr Σ θj (j = t - Lr + 1 to t)
REt = sign (log Ert/Eℓt), Trt = REt - REt-1
5626
IF THERE IS A RISE OR FALL BETWEEN -1 and +1 in Trt?
5628
NO
YES
ℓ = ℓ + 1
INDENTIFY tcℓ = t-1 as ℓth CHANGE TIME
5630
t = t + 1

FIG. 7

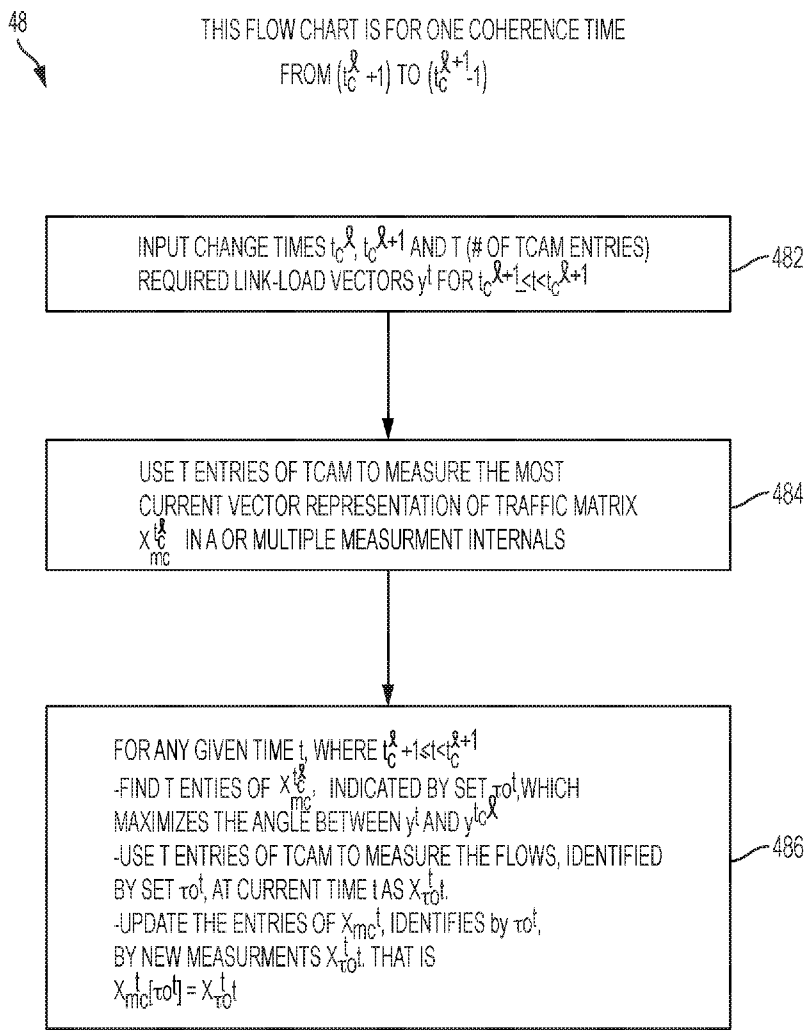

48
THIS FLOW CHART IS FOR ONE COHERENCE TIME FROM $(t_c^{\ell}+1)$ TO $(t_c^{\ell+1}-1)$
INPUT CHANGE TIMES t_c^{ℓ}, $t_c^{\ell+1}$ AND T (# OF TCAM ENTRIES) REQUIRED LINK-LOAD VECTORS y^t FOR $t_c^{\ell+1} \leq t < t_c^{\ell+1}$
482
USE T ENTRIES OF TCAM TO MEASURE THE MOST CURRENT VECTOR REPRESENTATION OF TRAFFIC MATRIX $X_{mc}^{t_c^{\ell}}$ IN A OR MULTIPLE MEASURMENT INTERNALS
484
FOR ANY GIVEN TIME t, WHERE $t_c^{\ell}+1 \leq t < t_c^{\ell+1}$
-FIND T ENTIES OF $X_{mc}^{t_c^{\ell}}$, INDICATED BY SET τo^t, WHICH MAXIMIZES THE ANGLE BETWEEN y^t AND $y^{t_c^{\ell}}$
-USE T ENTRIES OF TCAM TO MEASURE THE FLOWS, IDENTIFIED BY SET τo^t, AT CURRENT TIME t AS $X_{\tau o^t}^t$.
-UPDATE THE ENTRIES OF X_{mc}^t, IDENTIFIES by τo^t, BY NEW MEASURMENTS $X_{\tau o^t}^t$. THAT IS
$X_{mc}^t[\tau o^t] = X_{\tau o^t}^t$
486

FIG. 8

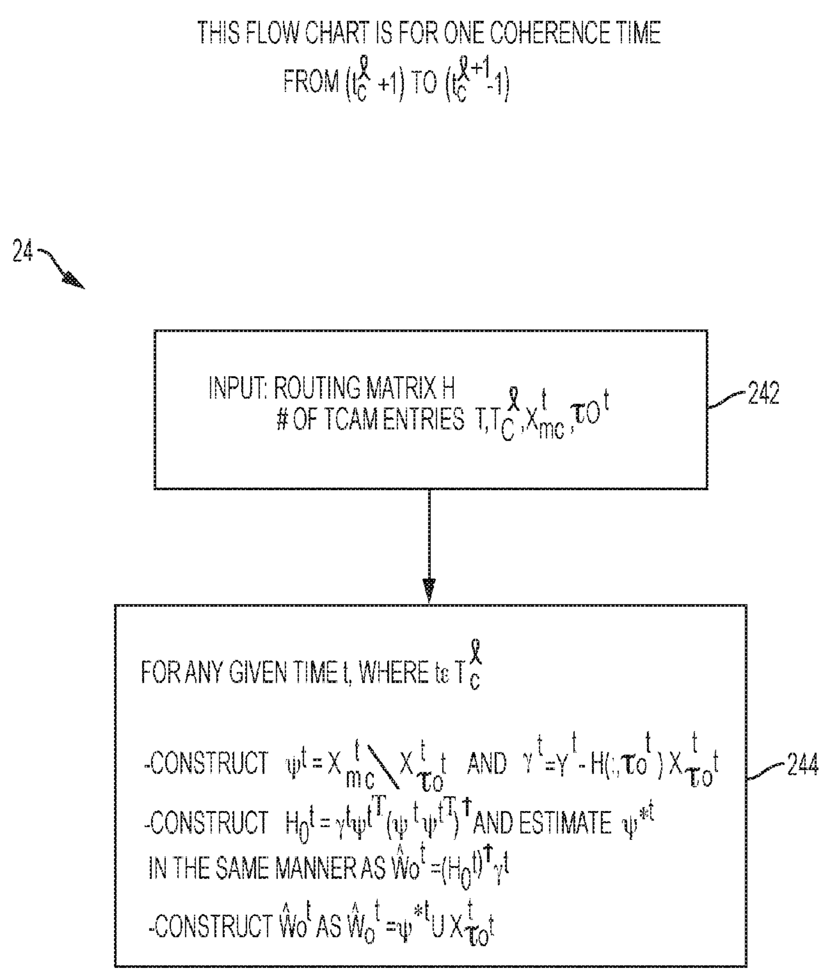

THIS FLOW CHART IS FOR ONE COHERENCE TIME
FROM $(t_c^{\ell}+1)$ TO $(t_c^{\ell+1}-1)$
24
INPUT: ROUTING MATRIX H
OF TCAM ENTRIES $T, T_C^{\ell}, X_{mc}^{t}, \tau o^{t}$
242
FOR ANY GIVEN TIME t, WHERE $t \varepsilon T_c^{\ell}$
-CONSTRUCT $\psi^t = X_{mc}^{t} \setminus X_{\tau o^t}^{t}$ AND $\gamma^t = Y^t - H(:, \tau o^t)\, X_{\tau o^t}^{t}$
-CONSTRUCT $H_0{}^t = \gamma^t \psi^{tT} (\psi^t \psi^{tT})^{\dagger}$ AND ESTIMATE ψ^{*t}
IN THE SAME MANNER AS $\hat{W}o^t = (H_0{}^t)^{\dagger} \gamma^t$
-CONSTRUCT $\hat{W}o^t$ AS $\hat{W}_0{}^t = \psi^{*t} \cup X_{\tau o^t}^{t}$
244

FIG. 9

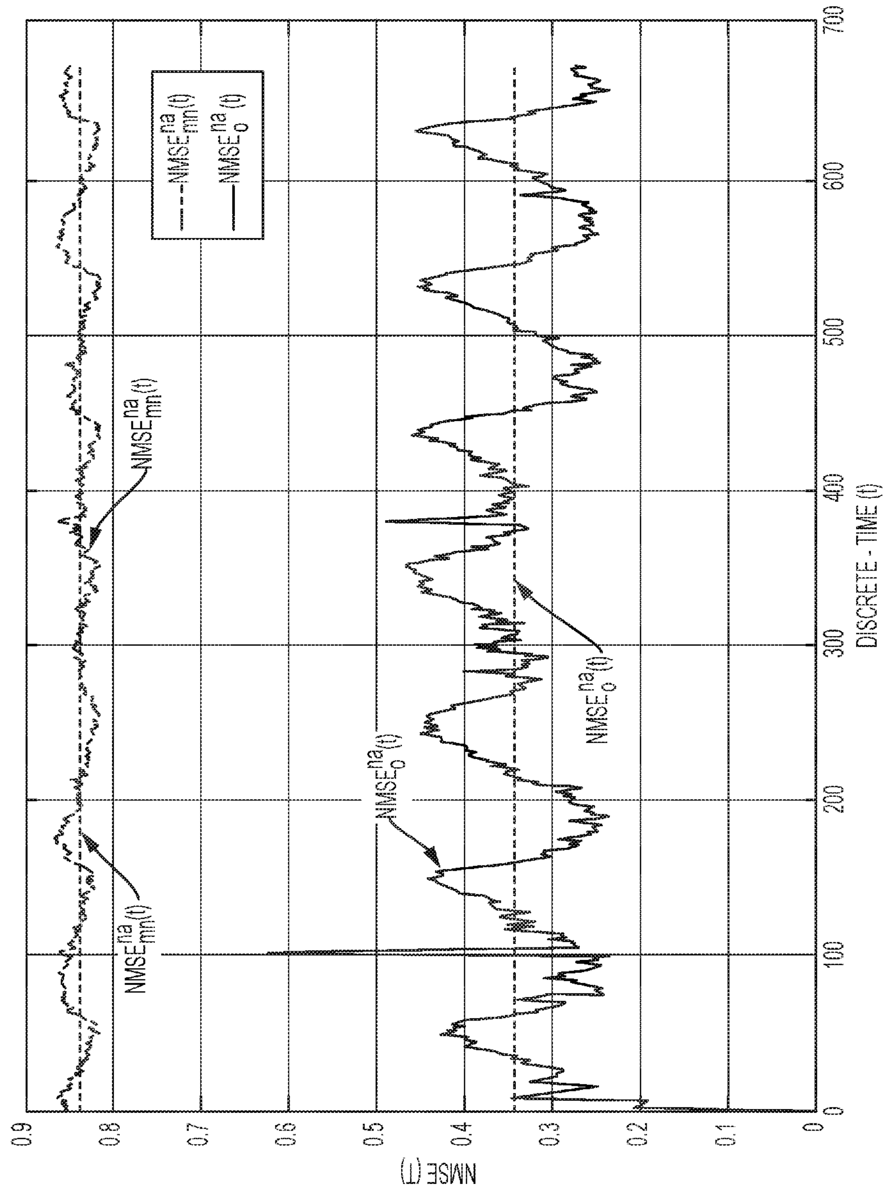
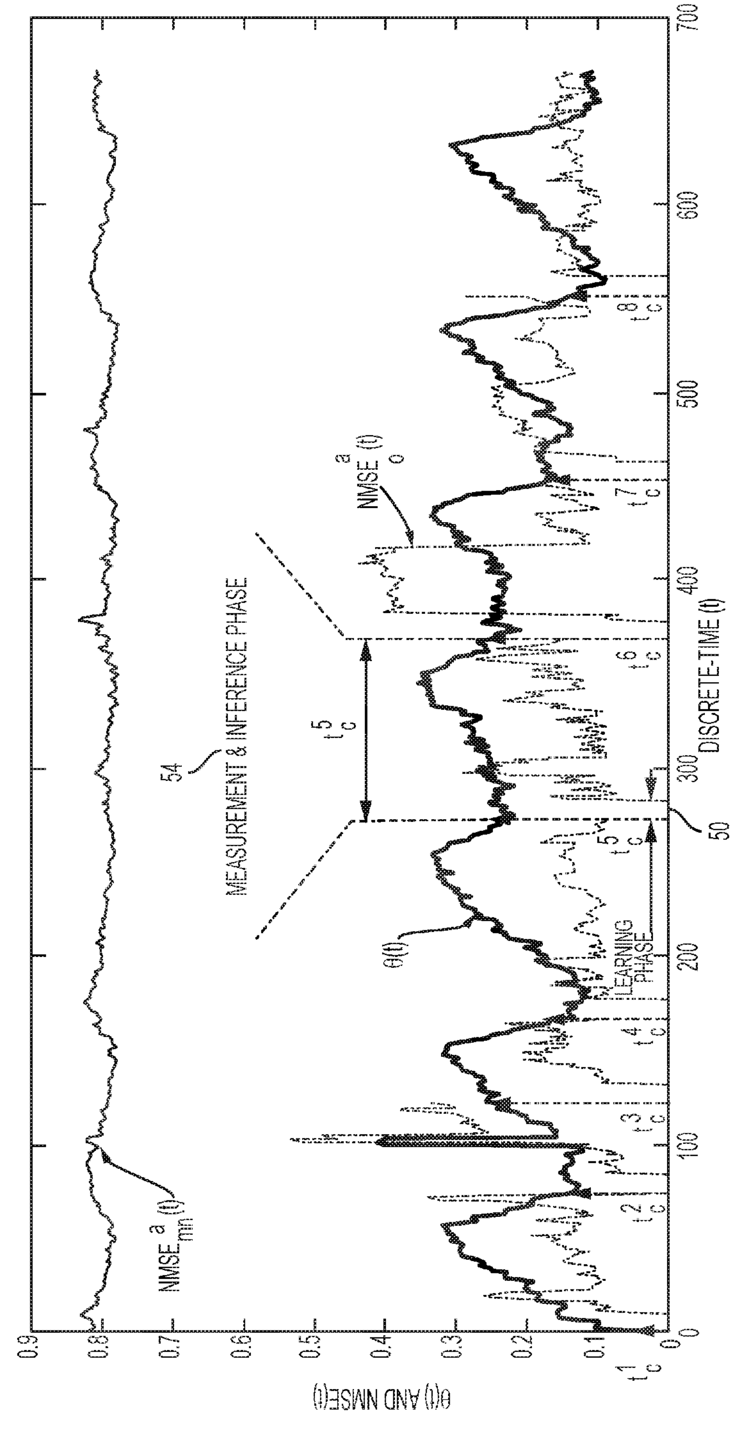

NMSE (T)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
DISCRETE - TIME (t)
0
100
200
300
400
500
600
700
$NMSE^{na}_{mn}(t)$
$NMSE^{na}_{o}(t)$
$NMSE^{na}_{mn}(t)$
$NMSE^{na}_{o}(t)$
$NMSE^{na}_{mn}(t)$
$NMSE^{na}_{o}(t)$

FIG. 10



θ(t) AND NMSE(t)
DISCRETE-TIME (t)
MEASUREMENT & INFERENCE PHASE
LEARNING PHASE
54
50
θ(t)
$NMSE^{a}_{mn}(t)$
$NMSE^{a}_{o}(t)$
t^{1}_{c}
t^{2}_{c}
t^{3}_{c}
t^{4}_{c}
t^{5}_{c}
t^{6}_{c}
t^{7}_{c}
t^{8}_{c}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
100
200
300
400
500
600
700

FIG. 11

SYSTEM FOR ESTIMATING UNKNOWN ATTRIBUTES OF INTEREST IN THE UNDER-DETERMINED INVERSE PROBLEM AND A PROCESS OF ACCOMPLISHING THE SAME

RELATED APPLICATION

This application claims priority to provisional patent application U.S. Ser. No. 62/105,437 filed on Jan. 20, 2015, the entire contents of which is herein incorporated by reference.

BACKGROUND

The embodiments herein relate generally to systems that control flow in systems.

Prior to embodiments of the disclosed invention there was no efficient way to solve an under-determined inverse problem. As a result, there was no way to produce optimized flows in an underdetermined system. Embodiments of the disclosed invention solve this problem.

SUMMARY

A method for solving an under-determined inverse problem or network inference/tomography problem in per-flow size, delay, loss and throughput inference in a computer network, through a system given an original observation matrix, includes the following steps, which are not necessarily in order. First, establishing the computer network having a plurality of nodes wherein the per-flow size, the delay, the loss and the throughput inferences are unknown; wherein an original routing matrix determines how flows from each node to every other node are appeared on links. Next, performing a learning phase by first measuring a set of quantities which are combinations of unknowns creating a first set of measurements. Then, measuring at least some of a set of unknowns creating a second set of measurements. After that, using the first set of measurements and the second set of measurements, an optimal observation matrix or pseudo-optimal observation matrix is computed. Following that, performing a computer controller adaptive measurement and inference phase by first measuring the set of quantities which are the combinations of the unknowns creating measurement quantities. Then, identifying a partial set of most informative unknown network flows that must be re-measured and updated. After that, re-measuring a partial set of unknowns. Next, re-computing and updating the optimal observation matrix to form an updated optimal observation matrix. Following that, estimating the set of unknowns using the measurement quantities, and a function of one of the set consisting of: the optimal observation matrix, the original observation matrix, or both.

In some embodiments the method can include determining a starting time of the learning phase and a duration of the learning phase by detecting sequential significant changes in measurements during the adaptive measurement and inference phase. Then, determining a set of most informative unknown network flows using at least one learning algorithm; wherein the at least one learning algorithm includes a multi-bandit algorithm.

In some embodiments, the method can include generating the optimal observation matrix with a regularized or non-regularized optimization technique including a least square estimation technique with or without constraints.

In some embodiments, the under-determined inverse problem is a traffic matrix estimation problem and a set of unknown network flows are estimated by first, measuring SNMP link loads as measured quantities which are linear combinations of a size of unknown network flows. Then, measuring a partial set of unknown network flow sizes using counters associated with one of the set consisting of: flow-table entries, TCAM entries or both. Next, computing the optimal observation matrix as a replacement of the original routing matrix. After that, inferring a set of unknown flows using SNMP link load measurements, and using specific estimation techniques that can include a function of one of the set consisting of the optimal observation matrix, the original routing matrix, or both.

A method for determining a size of network flows in a computer network through a system that can include the following steps which are not necessarily in order. First, providing a network having a plurality of nodes connected by a plurality of links, wherein nodes denote network devices and/or elements including computer hosts, physical switches, physical routers, virtual switches, and virtual routers. Next, performing a learning phase by iteratively performing the following steps until all link loads and a set of unknown network flow sizes have been measured, and an optimal observation matrix is computed. First, creating a measured link load vector by performing, first, measuring a load on a set of the plurality of links creating a plurality of link load measurements where link loads are linear combinations of a set of unknown network flows. Then, forming the measured link load vector from the plurality of link load measurements. After that, transferring the measured link load vector to a network controller connected to the computer network. Following that, creating a vector representation of a measured traffic matrix by measuring and storing a size of a flow from each node to every other node for a specified time period, by performing the following steps. First, installing a set of rules in one of a set consisting of: a storage system, a memory system, or both of one of the set consisting of: physical/virtual network switches, physical/virtual network routers, or both. Next, polling counters in the storage system that create flow-size measurements as flows are measured and network packets are routed. After that, transferring the measurements into a memory, and the network controller. Following that, forming the optimal observation matrix by taking a matrix multiplication of the measured link load vector, a transpose of the vector representation of the measured traffic matrix, and a pseudo inverse of the matrix multiplication of the vector representation of the measured traffic matrix and the transpose of the vector representation of the measured traffic matrix.

In some embodiments this method can include, performing an adaptive measurement and inference phase by iteratively and adaptively performing the following steps until adjusted link loads are measured, a set of most informative unknown network flows to be re-measured and updated is identified, the set of most informative unknown network flows are measured and the vector representation of a traffic matrix is updated to form an adjusted vector representation of the traffic matrix, an adjusted optimal observation matrix is computed, and an unknown vector representation of the traffic matrix is estimated. The steps include: first, establishing an iteration time. Next, starting a timer. After that, for a timer value less than the iteration time, performing the following steps. First, creating an adjusted link load vector by performing the following steps. First, measuring an adjusted load on the set of the plurality of links creating a plurality of adjusted link load measurements. Next, forming the adjusted link load vector from the plurality of adjusted link load measurements. After that transferring the adjusted link load vector to the network controller. Following that, creating the adjusted vector representation of the traffic matrix by storing and updating an adjusted size of the flow from a set of nodes to another set of nodes identified by the set of most informative unknown network flows by identifying the set of most informative unknown network flows that must be re-measured and updated. Then, installing an adjusted set of rules, for measuring identified most informative flows, in one of a set consisting of: the storage system, the memory system, or both of one of the set consisting of: the network switches, the network routers, or both. In some embodiments, the network switches can be either physical switches, virtual switches or both. In some embodiments, the network routers can be either physical routers, virtual routers or both. After that, polling adjusted counters in the one of a set consisting of: the storage system and the memory system that create adjusted measurements of the most informative flows as adjusted flows are measured and the network packets are routed. Following that, transferring the adjusted measurements into the memory and the network controller. Then, forming the adjusted vector representation of the traffic matrix by updating the size of the most informative flows. Following that, forming the adjusted optimal observation matrix by taking the matrix multiplication of the adjusted link load vector, the transpose of the adjusted vector representation of the traffic matrix, and the pseudo inverse of the matrix multiplication of the adjusted vector representation of the traffic matrix and the transpose of the adjusted vector representation of the traffic matrix. After that, creating an estimation of the vector representation of the traffic matrix by taking the matrix multiplication of the pseudo inverse of the adjusted optimal observation matrix and the adjusted link load vector.

In some embodiments, this method can include, detecting a significant change in the measurements. Then, restarting the learning phase. Finally, identifying the set of the most informative flows by finding a set of flows which maximize an angle between a current SNMP link-load vector and a SNMP link-load vector at a time of the significant change. In some embodiments, the storage system can be one of the set consisting of: TCAM, flow-table entries, or both. The network controller can be connected to switches and routers on the network through one of the set consisting of: a secure channel, a protocol, or both.

In some embodiments, an under-determined inverse problem is an inverse problem wherein the number of measurements is less than the number of unknowns that must be estimated. An over-determined inverse problem is the inverse problem wherein the number of measurements is larger than the number of unknowns that must be estimated. A square inverse problem is an inverse problem wherein the number of measurements is equal to the number of unknowns that must be estimated.

BRIEF DESCRIPTION OF THE FIGURES

The detailed description of some embodiments of the invention is made below with reference to the accompanying figures, wherein like numerals represent corresponding parts of the figures.

FIG. **1** shows a schematic view of one embodiment of the present invention;

FIG. **2** shows a schematic view of one embodiment of the present invention;

FIG. **3** shows a schematic view of one embodiment of the present invention;

FIG. **4** shows a schematic view of one embodiment of the present invention;

FIG. **5**A shows a schematic view of one embodiment of the present invention;

FIG. **5**B shows a schematic view of one embodiment of the present invention;

FIG. **5**C shows a schematic view of one embodiment of the present invention;

FIG. **5**D shows a schematic view of one embodiment of the present invention;

FIG. **5**E shows a schematic view of one embodiment of the present invention;

FIG. **6**A shows a schematic view of one embodiment of the present invention;

FIG. **6**B shows a schematic view of one embodiment of the present invention;

FIG. **6**C shows a schematic view of one embodiment of the present invention;

FIG. **6**D shows a schematic view of one embodiment of the present invention;

FIG. **6**E shows a schematic view of one embodiment of the present invention;

FIG. **6**F shows a schematic view of one embodiment of the present invention;

FIG. **7** shows a schematic view of one embodiment of the present invention;

FIG. **8** shows a schematic view of one embodiment of the present invention;

FIG. **9** shows a schematic view of one embodiment of the present invention;

FIG. **10** shows a graphical output of one embodiment of the present invention;

FIG. **11** shows a graphical output of one embodiment of the present invention;

DETAILED DESCRIPTION OF CERTAIN EMBODIMENTS

By way of example, and referring to FIG. **1**, a controller **10** is communicatively coupled to an operating network **20** using a network communication system including network measurement controlling messages **14**. Operating network **20** further comprises software defined network devices and non-software defined network devices. Network measurement controlling messages **14** can dynamically configure or reconfigure the software defined measurement network **12** as well as poll required network measurements and statistics **16**. Operating network **20** and software defined measurement network **12** further comprise a plurality of network devices and elements including open flow switches **18**, and simple network management protocol (SNMP) agents. Controller **10**, which can either be a physical component or a virtualized controller, further comprises SNMP management system **36** and network configuration protocol. Software defined measurement network **12** further comprises a plurality of open flow switches **18**A, **18**B, **18**C, **18**D, and **18**E in an operating network **20**. The plurality of open flow switches **18**A, **18**B, **18**C, **18**D, and **18**E are either mainly or partially for network traffic measurement and network packet forwarding and routing. In some embodiments, the plurality of open flow switches can be one of the set consisting of physical switches, virtual switches or both. Controller **10** further comprises an adaptive per flow measurement module **22** and an optimal-coherent inference module **24**. The adaptive per flow measurement module **22** indicates the set of most informative flows **44** in the operating network **20** that can be directly measured. The optimal-coherent inference module **24** processes the measurements and statistics **16** and provides accurate estimates of network flows **42**. The controller **10**, then sends network measurement controlling messages **14** to the software defined measurement network **12**. In some embodiments, required per flow measurements **32** and aggregated flow measurements **32** are measured by NetFlow protocol, sFlow protocol, and probing packets.

The network measurement controlling messages **14** indicate which flow in the operating network **20** should be directly measured at different times or locations by reconfiguring either a ternary content addressable memory **26**, a flow table or both. The ternary content addressable memory **26** (TCAM) models the TCAM entries of the plurality of open flow switches **18**A, **18**B, **18**C, **18**D, and **18**E. The flexibility of open flow is used to install TCAM wildcard matching rules or prefix keys **28** and collect associated statistics **30**. Associated statistics **30** can include per flow measurements **32** or aggregated measurement of multiple flows in each measurement interval **40**. The longest prefix matching forwarding table mechanism in open flow switches **18** facilitates the implementation of a per-flow measurement process where prefix keys **28** of the per-flow measurements **34** are stored in the ternary content addressable memory **26** or flow tables and corresponding counters measure the size of flows. This process is called de-aggregation.

In some embodiments, controller **10** further comprises a SNMP management system **36**, alternately, SNMP management system **36** is communicatively coupled to controller **10**. Network measurement controlling messages **14** are a mean of the interaction of the Simple Network Management Protocol (SNMP) management system **36** and SNMP agents **38** to measure SNMP link-loads. In some embodiments, controller **10** and SNMP management system **36** are collocated in one machine and in other embodiments, they are located in separate machines and they can poll the statistic counts periodically or in different measurement intervals the frequency of which is limited by practical constraints. Regardless, a focus is on traffic matrix **52** measurement and inference where ternary content addressable memory **26** entries of all of the open flow switches **18** of the software defined measurement network **12** is modeled as a large flow table.

FIG. **2** shows another view of operating network **20**. At each measurement interval **40** or τ there are m SNMP link load measurements defined by the vector $Y_{m\times 1}^t:=\{y_i^t\}_{i=1}^m$ and $n=N(N-1)$ flows which form the vector representation of traffic matrix **52**, here $X_{n\times 1}^t:=\{x_j^t\}_{j=1}^n$. The SNMP link load vector **56** or Y^t is linearly related to the traffic matrix **52**, here X^t as $Y^t=HX^t$ where j^{th} 1 in i^{th} row of the binary routing matrix H represents the contribution of the j^{th} flow x_j^t in the i^{th} link load y_i^t. If H is not binary, then j^{th} value in i^{th} row of matrix H represents the contribution of the j^{th} attribute x_j^t in the i^{th} quantity y_i^t. The set of origin-destination flows in network exhibit a constant behavior in the sense that the dynamic of network flows **42** are predictive over time intervals with different durations, called as coherence time. The notion of coherence time is the time interval within which network flows **42** demonstrate, on average, a consistent and predictable behavior. Additionally, coherence time is defined as the time between the occurrences of two consecutive major changes in the behavior of the traffic matrix **52** which is quantified by an appropriate metric.

FIG. **3** represents the behavior of a network flow over multiple coherence times in the software defined measurement network **12**. Change time t_c^l indicates the time of the l^{th} major change in the network traffic and the 1^{th} coherence time is the interval between 1^{th} and $(l+1)^{th}$ major change times in network flows **42**. That is, $T_c^l=\{t_c^l+1, \ldots, t_c^{l+1}-1\}$.

Controller **10** assumes that at the t^{th} measurement interval **40** with measurement interval **40**, two sets of network measurements are provided. First the SNMP management system **36** interacts with SNMP agents **38** and provides the SNMP link load measurements Y^t. Second, under hard resource constraint of the size of ternary content addressable memory **26** in flow tables of open flow switches **18**A, **18**B, **18**C, **18**E where the sum of all ternary content addressable memories in switches $T \ll n$, all T entries in ternary content addressable memory **26** are used to track and measure all flows and/or the set of most informative flows **44**A, **44**B and **44**C and to provide T per flow measurements **32**.

Returning to FIG. **2**, adaptive per flow measurement module **22** comprises two methods: change detection method **46** and adaptive traffic matrix updating method **48**. At a first measurement interval **40** in the beginning of the learning phase **50**, a first SNMP link-load vector $Y^{t_c^l}$ is created. At t^{th} measurement interval **40**, a second SNMP link-load vector Y^t is created. Additionally, the SNMP link-load vector Y^t is compared with the first SNMP link-load vector $Y^{t_c^l}$ with change detection method **46**. The detection of the l^{th} significant change in Y^ts indicates the beginning of a new coherence time and the beginning of a learning phase **50**. At the new coherence time t_c^l, a learning phase **50** begins. In a learning phase **50**, all available T entries of the ternary content addressable memory **26** in the software defined measurement network **12** are used to measure the most current traffic matrix **62**. In a computer controlled measurement and inference phase **54**, that is, over l^{th} coherence time T_c^l, the T most informative flows **44**A, **44**B and **44**C are directly measured and T flows **42** are updated in the most current traffic matrix **62**.

Controller **10** further utilizes the optimal-coherent inference module **24**. Optimal-coherent inference module **24** uses the most current traffic matrix **62** and the SNMP link load vector **56** to identify an optimal observation matrix **58** of SNMP link load vectors **56**, denoted by H_O^t. At each measurement interval **40**, the per flow measurements **32** and the SNMP link-loads are used to coherently estimate the traffic matrix **60** or $\hat{X}_O^t$ (equivalently $\hat{W}_O^t$) using the following optimization formulation represented in the equation below where $X_{\tau 0^t}^t$ denotes the set of directly measured per-flow counts,

$$X^t_{\overline{\tau 0^t}}$$

denotes the set of flows that must be estimated, and

$$X^t = X^t_{\tau 0^t} \cup X^t_{\overline{\tau 0^t}}.$$

$$\hat{W}_O^t = \min_{X^t_{\overline{\tau 0^t}}} \left\| X^t_{\overline{\tau 0^t}} \right\|_2$$

$$\text{s.t. } X^t_{\tau 0^t} \text{ is known and } Y^t = H_O^t X^t \text{ and } X^t_{\tau 0^t} \cap X^t_{\overline{\tau 0^t}} \neq \emptyset$$

Under hard constraints of limited resources in software defined measurement network **12** this framework is simple, flexible and efficient with very high estimation accuracy which can be deployed on the plurality of open flow switches **18**A, **18**B, **18**C, **18**D, and **18**E. Moreover, the communication overhead between controller **10**, and the plurality of open flow switches **18**A, **18**B, **18**C, **18**D, and **18**E is low because the required measurements for the fine-grained estimation of network flows **42** are aggregated SNMP link loads which are readily and reliably available in ISP networks. Further, only a few most beneficial flows are required to be directly measured.

FIG. **4** conceptually illustrates an electronic system **200** with which some embodiments of the invention are implemented. The electronic system **200** may be a computer, phone, PDA, or any other sort of electronic device. Such an electronic system includes various types of computer readable media and interfaces for various other types of computer readable media. Electronic system **200** includes a bus **205**, processing unit(s) **210**, a system memory **215**, a read-only **220**, a permanent storage device **225**, input devices **230**, output devices **235**, a network **240**, a controller **250**, and measurement devices **260**.

The bus **205** collectively represents all system, peripheral, and chipset buses that communicatively connect the numerous internal devices of the electronic system **200**. For instance, the bus **205** communicatively connects the processing unit(s) **210** with the read-only **220**, the system memory **215**, and the permanent storage device **225**.

From these various memory units, the processing unit(s) **210** retrieves instructions to execute and data to process in order to execute the processes of the invention. The processing unit(s) may be a single processor or a multi-core processor in different embodiments.

The read-only-memory (ROM) **220** stores static data and instructions that are needed by the processing unit(s) **210** and other modules of the electronic system. The permanent storage device **225**, on the other hand, is a read-and-write memory device. This device is a non-volatile memory unit that stores instructions and data even when the electronic system **200** is off. Some embodiments of the invention use a mass-storage device (such as a magnetic or optical disk and its corresponding disk drive) as the permanent storage device **225**.

Other embodiments use a removable storage device (such as a floppy disk or a flash drive) as the permanent storage device **225**. Like the permanent storage device **225**, the system memory **215** is a read-and-write memory device. However, unlike storage device **225**, the system memory **215** is a volatile read-and-write memory, such as a random access memory. The system memory **215** stores some of the instructions and data that the processor needs at runtime. In some embodiments, the invention's processes are stored in the system memory **215**, the permanent storage device **225**, and/or the read-only **220**. For example, the various memory units include instructions for processing appearance alterations of displayable characters in accordance with some embodiments. From these various memory units, the processing unit(s) **210** retrieves instructions to execute and data to process in order to execute the processes of some embodiments.

The bus **205** also connects to the input and output devices **230** and **235**. The input devices enable the person to communicate information and select commands to the electronic system. The input devices **230** include alphanumeric keyboards and pointing devices (also called "cursor control devices"). The output devices **235** display images generated by the electronic system **200**. The output devices **235** include printers and display devices, such as cathode ray tubes (CRT) or liquid crystal displays (LCD). Some embodiments include devices such as a touchscreen that functions as both input and output devices.

Bus **205** also couples electronic system **200** to a network **240** through a network adapter (not shown). In this manner, the computer can be a part of a communications network, or a network of computers (such as a local area network ("LAN"), a wide area network ("WAN"), or an intranet), or a network of networks (such as the Internet). Any or all components of electronic system **200** may be used in conjunction with the invention.

Bus **205** couples electronic system **200** to controller **250** and measurement devices **260**. In some embodiments controller **250** can adaptively determine a set of required quantities and unknowns of interest that must be measured at important instances of time. Measurement devices **260** can be used to measure a set of required quantities and unknowns of interest.

These functions described above can be implemented in digital electronic circuitry, in computer software, firmware or hardware. The techniques can be implemented using one or more computer program products. Programmable processors and computers can be packaged or included in mobile devices or embedded systems. The processes may be performed by one or more programmable processors and by one or more set of programmable logic circuitry. General and special purpose computing and storage devices can be interconnected through communication networks.

Some embodiments include electronic components, such as microprocessors, storage and memory that store computer program instructions in a machine-readable or computer-readable medium (alternatively referred to as computer-readable storage media, machine-readable media, or machine-readable storage media). Some examples of such computer-readable media include RAM, ROM, read-only compact discs (CD-ROM), recordable compact discs (CD-R), rewritable compact discs (CD-RW), read-only digital versatile discs (e.g., DVD-ROM, dual-layer DVD-ROM), a variety of recordable/rewritable DVDs (e.g., DVD-RAM, DVD-RW, DVD+RW, etc.), flash memory (e.g., SD cards, mini-SD cards, micro-SD cards, etc.), magnetic and/or solid state hard drives, read-only and recordable Blu-Ray® discs, ultra density optical discs, any other optical or magnetic media, and floppy disks. The computer-readable media may store a computer program that is executable by at least one processing unit and includes sets of instructions for performing various operations. Examples of computer programs or computer code include machine code, such as is produced by a compiler, and files including higher-level code that are executed by a computer, an electronic component, or a microprocessor using an interpreter.

Optimal-coherent inference module **24** contains the following steps. First, at the l^{th} time change t_c^l, the optimal observation matrix **58** of SNMP link load vectors **56** is identified as $H_O^{t_c^l}$. This can be done by first pooling the SNMP link load vectors **56** at $t=t_c^l$ using SNMP management system **36** and measuring the most current traffic matrix **62** at $t=t_c^l$. Next, a minimum norms solution is computed using the optimal observation matrix **58** for $t=t_c^l+1:1:t_c^{l+1}-1$. Both of these stages involve underdetermined inverse problems because all m×n elements of the optimal observation matrix **58** must be measured using m+n measurements of SNMP link load vector **56** and traffic matrix **52** at $t=t_c^l$ and $m+n<m\times n$. In the second stage, m per-flow measurements **32** is less than n flows **42**.

$$\text{In Stage 1}: H_o^t = \underset{H^t}{\operatorname{argmin}} \|Y^t - H^t X^t\|_2^2 \text{ at } t = t_c^l$$

$$\text{In Stage 2}: \hat{W}_O^t = \underset{X^t}{\operatorname{argmin}} \|X^t\|_2 \; s.t. \; Y^t = H^t X^t \text{ for } t \in = T_c^l$$

$$\text{In Stage 3}: \hat{W}_O^t = H_o^{t\dagger} Y^t \text{ where } H_o^{t\dagger} := \left(H_o^{tT}(H_o^t H_o^{tT})^{-1}\right)$$

Where † indicates the pseudo-inverse operator, T indicates the complex conjugate transpose of the matrix, and $\hat{W}_O^t$ denotes the estimate of the vector representation of the traffic matrix **52** or X^t at time t.

The optimal observation matrix **58** in Stage 1 can be computed as $H_o^t = Y^t X^{t^T}(X^t X^{t^T})^\dagger$. In some embodiments, the general form of optimal observation matrix **58** or H_o^t can be used as $H_{G^o}^t = (Y^t X^{t^T}(X^t X^{t^T})^\dagger + QX^\perp)$ where Q is an arbitrary m×m matrix and $X^\perp$ is the space of the rows perpendicular to X^t. The inverse or pseudo-inverse of a matrix can be computed using different techniques including the singular value decomposition method.

In the learning phase, Minimum norm estimations using $H_o^t = Y^t X^{t^T}(X^t X^{t^T})^\dagger$ has several properties. First, estimations are identifiable, that is, $\hat{W}_O^t = (H_o^t)^\dagger Y^t = X^t$. Second, measurements are compatible, that is, $Y^t = H_o^t X^t$. Third, estimations are noise immune because even if measurement vector Y^t is noisy, then $\hat{W}_O^t = (H_o^t)^\dagger Y^t = X^t$.

In some embodiments, $\hat{W}_O^t$ can be computed using both the original observation matrix H and the optimal observation matrix **58** or H_o^t, including the minimum-norm estimation technique as follows:

$$\hat{W}_O^t = \left(\begin{bmatrix} H \\ H_o^t \end{bmatrix}\right)^\dagger \begin{bmatrix} Y^t \\ Y^t \end{bmatrix}.$$

In some embodiments, optimal-coherent inference module **24** can be alternately computed using regularized estimation techniques including the minimization of the addition of $\|Y^t - H^t X^t\|$ and a fraction of $\|H^t\|$. The unknown in this minimization problem that must be estimated is H^t, and operator norm $\|.\|$ is appropriately defined based on the application.

In some embodiments, computing the optimal observation matrix **506** can alternately include the following steps. Given a set of original observations, a set of new measurements is provided by designing the optimal observation matrix **506** which provides a set of most informative measurements to the set of given observations. This set of new observations is complementary to the set of given original observations and tries to minimize the estimation error as much as possible. For the application in network inference, let Y_{SS} be a set of T_{SS} vectors of original given link-loads Y^t where Y^t is an m×1 vector for t=1, . . . , T_{SS}. At each time interval t, $Y^t = HX^t$ where H is the original routing matrix with size m×n, and X^t is the vector representation of the traffic matrix **52**. X^t is assumed to be known over T_{SS} periods of time t=1, . . . , T_{SS} in the learning phase **50**. Then, a row of the optimal observation matrix **58** or H_{On} with n columns can be designed by maximizing distance d_d^2, defined in the equation below:

$$h_{on} = \underset{h_{on}}{\operatorname{argmax}} d_d^2 = \underset{h_{on}}{\operatorname{argmax}} (h_i X^t - h_{on} X^t)^2 \text{ for } i \in \{1, \ldots, m\} \text{ and } t \in \{1, \ldots, T_{SS}\}$$

Here, h_i denotes the i^{th} row of matrix H, X^t denotes the t^{th} vector representation of the traffic matrix **52**, and h_{on} denotes a row of matrix H_{on}. This maximization can be performed, in an iterative way, using the least square estimation approach, including the Least Mean Squares (LMS) algorithm, defined in the equation below:

$$h_{on}(j+1) = h_{on}(j) - 2\mu d X^{t^T}$$

Here, there is iteration over parameter j until convergence. X^{t^T} is the complex conjugate transpose of X^t. μ is set to a small positive real number less than one. h_{on} is normalized by its norm $\|h_{on}\|$ at each iteration j. Accordingly, by designing m_1 rows, an optimal observation matrix **58** or H_{On} with size m_1×n can be designed. A function of H_{on}, denoted by H_{on}^f, can be used to collect a new set of measurements $Y_n^t = H_{on}^f X^t$ where X^t is the unknown vector representation of the traffic matrix **52** for $t > T_{SS}$. An estimate of X^t, can be inferred by computing the matrix multiplication of the pseudo-inverse of H_{on}^f and Y_n^t.

This process can include the following steps of learning phase **50**. First, measuring a set of vector of quantities, which are combinations of unknowns, over several period of times and creating a first set of measurements. Next, measuring a set of vector of unknowns over several period of times and creating a second set of measurements. Finally, computing one of the set consisting of an optimal observation matrix **506** and pseudo-optimal observation matrix by using the first set of measurements and the second set of measurements, and the least square estimation approach, including the Least Mean Squares (LMS) algorithm.

FIG. **5**A shows one embodiment of learning phase **50** in more detail. Learning phase **50** comprises the following steps, which are not necessarily in order. First, at step **502**, measuring a set of quantities which are combinations of unknowns creating a first set of measurements. Next, at step **504**, measuring at least some of a set of unknowns creating a second set of measurements. After that, at step **506**, using the first set of measurements and the second set of measurements to form the optimal observation matrix **58** or pseudo-optimal observation matrix. In some embodiments, this can be done with a regularized optimization technique including a least square estimation technique with constraints. The regularized optimization technique can include the least square estimation technique without the constraints. Alternately, this can be done with a non-regularized optimization technique including a least square estimation technique with constraints. The non-regularized optimization technique can include the least square estimation technique without the constraints. Additional combinations of these can exist.

Turning to FIG. **5**B, in some applications such as an under-determined multiple-input multiple output (MIMO) communication system, step **506** can be shown in more detail. In the learning phase **50**, for all possible vectors of communication symbols, the optimal observation matrix **58** can be computed as follows. First, at step **5062**, transmitting a vector of known symbols at multiple transmitters. Next, at step **5064**, measuring the vector of known symbols at multiple receivers, and thereby creating vectors of measurements. After that, at step **5066**, computing the optimal observation matrix **58** for each vector of known symbols creating a plurality of optimal observation matrixes **58**.

Finally, saving the vector of known symbols and the plurality of optimal observation matrix **58** at a receiver device.

Turning to FIG. **5**C, learning phase **50** can be completed by performing the following steps. First, at step **508**, creating a measured link load vector. Then, at step **510**, creating a vector representation of a measured traffic matrix **52** by measuring and storing a size of a flow from each node to every other node for a specified time period. After that, at step **512**, forming the optimal observation matrix **58** by taking a matrix multiplication of the measured link load vector, a transpose of the vector representation of the measured traffic matrix **52**, and a pseudo inverse of the matrix multiplication of the vector representation of the measured traffic matrix **52** and the transpose of the vector representation of the measured traffic matrix **52**.

Turning to FIG. **5**D, in some embodiments, step **508**—creating a measured link load vector can be accomplished with the following steps, which are not necessarily in order. First, at step **5082**, measuring a load on a set of the plurality of links creating a plurality of link load measurements where link loads are linear combinations of unknown network flows **42**. Next, at step **5084**, forming the measured link load vector from the plurality of link load measurements. After that, at step **5086**, transferring the measured link load vector to a network controller **10** that is connected to the computer communications network.

Turning to FIG. **5**E, in some embodiments, step **510** creating a vector representation of a measured traffic matrix **52** by measuring and storing a size of a flow from each node to every other node for a specified time period can be accomplished with the following steps which are not necessarily in order. First, at step **5012**, installing a set of rules in one of a set consisting of: a storage system, a memory system, or both of one of the set consisting of: network switches, network routers, or both. After that, at step **5014**, polling counters in the storage system that create flow-size measurements as flows are measured and network packets are routed. Following that, at step **5016**, transferring the measurements into a memory, and the network controller **10**.

Turning to FIG. **6**A, in some embodiments, computer controlled adaptive measurement and inference phase **54** includes the following steps, which are not necessarily in order. First, at step **542**, measuring the set of quantities which are the combinations of the unknowns creating measurement quantities. Next, at step **544**, identifying a partial set of most informative unknown network flows **42** that must be re-measured and updated. Following that, at step **546**, re-measuring a partial set of unknowns. After that at step **548**, re-computing and updating the optimal observation matrix **58** to form an updated optimal observation matrix **58**. Then, at step **550**, estimating the set of unknowns using the measurement quantities, and a function of one of the set consisting of: the optimal observation matrix **58**, the original observation matrix, or both.

In FIG. **6**B, step **5420**, in some embodiments, the computer controlled measurement and inference phase **54** includes the following steps, which are not necessarily in order. First, at step **5422**, transmitting an unknown vector of symbols from multiple transmitters. Next, at step **5424**, creating a measurement vector from multiple measurements at multiple receivers. Following that, at step **5426**, estimating vectors of symbols using the measurement vector and the optimal observation matrices creating estimated vectors of symbols. Then, at step **5428**, computing a distance between the estimated vectors of symbols, and the vectors of known symbols. After that, at step **5430**, decoding a minimum distance vector of known symbols as an ultimate estimate of the unknown vector of symbols.

In FIG. **6**C, at step **550**, estimating the set of unknowns using the measurement quantities, further includes the following steps which are not necessarily in order. First, at step **5502**, measuring SNMP link loads as measured quantities which are linear combinations of a size of unknown network flows **42**. Then, at step **5504**, measuring a set of unknown network flow sizes **30** using counters associated with one of the set consisting of: flow-table entries, TCAM entries or both. After that, at step **5506**, computing the optimal observation matrix **58** as a replacement of the original routing matrix. Following that, at step **5508**, a set of unknown flows are inferred using specific estimation techniques and using SNMP link load measurements, and the function of one of the set consisting of the optimal observation matrix **58**, the original routing matrix, or both.

In FIG. **6**D, one embodiment of computer controlled measurement and inference phase **54**, includes the following steps, which are not necessarily in order. First, at step **552**, establishing an iteration time. Next, at step **554**, starting a timer. Following that, at step **556**, for a timer value less than the iteration time, performing the following steps. If not, then stop the process at step **574**.

If the process continues, at step **558**, creating an adjusted link load vector. Then, at step **560**, transmitting and storing the adjusted link load vector to the controller **10**. After that, at step **562**, detecting a significant change in the measurement quantities. If so, after that, at step **570**, restarting the learning phase **50**.

If not, then, at step **572**, identifying a partial set of most informative flows **44**. Following that, at step **576**, measuring and storing a partial set of most informative flows by installing an adjusted set of rules and polling counters. Next, at step **578**, transmitting adjusted measurements to the controller **10** and memory. After that at step **580**, forming the adjusted optimal observation matrix **58** by taking the matrix multiplication of the adjusted link load vector, the transpose of the adjusted vector representation of the traffic matrix **52**, and the pseudo inverse of the matrix multiplication of the adjusted vector representation of the traffic matrix **52** and the transpose of the adjusted vector representation of the traffic matrix **52**. Following that, at step **582**, creating an estimation of the vector representation of the traffic matrix **52** by taking the matrix multiplication of the pseudo inverse of the adjusted optimal observation matrix **58** and the adjusted link load vector.

In FIG. **6**E, step **558**, creating an adjusted link load vector, can include the following steps, which are not necessarily in order. First, at step **5582** measuring a load on a set of the plurality of links creating a plurality of link load measurements where link loads are linear combinations of unknown network flows **42**. After that, at step **5584**, forming the measured link load vector from the plurality of link load measurements. Following that, at step **5586**, transferring the measured link load vector to a network controller **10** connected to the computer network.

In FIG. **6**F, step **585**, creating the adjusted vector representation of the traffic matrix **52**, can be accomplished with the following steps, which are not necessarily in order. First, at step **5822** installing a set of rules in one of a set consisting of: a storage system, a memory system, or both of one of the set consisting of: network switches, network routers, or both. Next, at step **5824**, polling counters in the storage system that create flow-size measurements as flows are measured and network packets are routed. After that, at step **5826** transferring the measurements into a memory and the network controller **10**.

The adaptive per flow measurement module **22** has two main components, the change detection method **46** and adaptive traffic matrix updating method **48**. The change in the network traffic is measured by computing the angle between two SNMP link-load vectors, defined in the equation:

$$\theta^t = \arccos\frac{\langle Y^t, Y^{t_c^l}\rangle}{\|Y^t\|_2\|Y^{t_c^l}\|_2} \text{ where } t \in T_c^l \text{ and } 0 \le \theta^t \le \frac{\pi}{2}$$

Where Y^t and $Y^{t_c^l}$ denote the SNMP link-load vectors at the beginning of the l^{th} coherence time and at the current time t, respectively. That is, θ^t quantifies the deviation of two SNMP link-load vectors at two different times Y^t and $Y^{t_c^l}$ as an indication of a major change in the network traffics. Accordingly and to have a causal change detection method, at any time t, the average of θ^t's over two periods of time, determined by two windows w_t^l and w_t^r with lengths L_l and L_r, is compared where $w_l^t:=\{t-L_l-L_r+1, \ldots, t-L_r\}$ and $w_r^t:=\{t-L_r+1, \ldots, t\}$. The transition between two consecutive relative comparisons is reported as a change time t_c^l. Turning to FIG. **7**, The pseudo-code of this process step **562**, detecting a significant change in the measurement quantities is as follows:

Input: $\theta=\{\theta^t\}_{t=1}^t$, w_l^t, w_r^t at step **5622**

Output: The set of detected change times $S_c^t:=\{t_c^1, t_c^2, \ldots\}$

Initialization: $l=0$, $S_c^t=\emptyset$

While $t>L_l+L_r+1$ do at step **5624**

$$E_l^t = \frac{1}{L_l}\sum_{j=t-L_l-L_r+1}^{t-L_r}\theta^j,\ E_r^t = \frac{1}{L_r}\sum_{j=t-L_r+1}^{t}\theta^j \text{ and}$$

$$R_E^t = \operatorname{sign}\left(\log\frac{E_r^t}{E_l^t}\right),\ T_r^t = R_E^t - R_E^{t-1} \text{ at step 5626}$$

If there is a rise or fall between −1 and +1 in T_r^t at step **5628** then

$l=l+1$, $t_c^l=t-1$, $S_c^t(l)=t_c^l$ at step **5630**

End if

End while

This method is not based on any statistical assumption about θ_t, and thus, a more change detection method **46** can be used to enhance its performance. Without loss of generality and to simplify the current worst-case implementation of the change detection method **46**, θ_t is computed as the angle between Y_t and Y_1 for $t\in[2, \ldots, T_0]$ where T_0 is the duration of traffic in each data set.

Turning to FIG. **8**, to adaptively update the most current traffic matrix **62** or X_{mc}^t, which is required for the optimal-coherent inference module **24**, the flexibility of the software defined measurement network **12** is used to allocate the limited available ternary content addressable memory **26** entries for measuring the most informative flows **44** that leads to the best possible estimation accuracy using the optimal-coherent inference module **24**. The following method summarizes the adaptive traffic matrix updating method **48** where by detecting the l^{th} change time t_c^l all T entries of ternary content addressable memory **26** are used to measure the $X_{mc}^t(:=X^{t_c^l})$ in one or more

$$\left\lceil\frac{n}{T}\right\rceil$$

measurement epochs; in this case, $\tau0^t=\{1, \ldots, n\}$ and $|\tau0^t|=n$.

Input: number of TCAM entries T, l^{th} change time t_c^l, $(l+1)^{th}$ change time t_c^{l+1} at step **472**

Output: At each epoch t, the updated traffic matrix **52** or X_{mc}^t and $\tau0^t$

Initialization: Use T entries of TCAM to measure $X_{mc}^t:=X_{mc}^{t_c^l}$ at step **484**

While $t_c^l+1\le t\le t_c^{l+1}$, do at step **486**

Find T entries of the updated traffic matrix **52** or X_{mc}^t indicated by the set $\tau0^t$, which maximizes the angle between Y^t and $Y^{t_c^l}$.

Use T entries of the TCAM to measure the indicated flows at the current time t as $X_{\tau0}^t$

$X_{mc}^t[\tau0^t]=X_{\tau0}^t$, which in MATLAB language means replacing all elements of X_{mc}^t with indices in the set $\tau0^t$ by $X_{\tau0}^t$.

End while

As shown in FIG. **9**, for $t_c^l+1\le t\le t_c^{l+1}$ the T flows **42** which maximize θ_t are directly measured. These per flow measurements **34** have the most contribution in equalizing/coinciding the two SNMP link-load vectors Y^t and $Y^{t_c^l}$; in this case, $\tau0^t=\{i_1, \ldots i_j, \ldots, i_T\}$ where $i_j\in\{1, \ldots, n\}$ and $|\tau0^t|=T$. Accordingly, at each epoch t, also, T elements of traffic matrix **52** vector X_{mc}^t are updated by the recently measured flows **42**. The following process represents the main steps in implementing the optimal-coherent inference module **24** in a sample coherence time T_c^l.

Input: Routing matrix H, # of TCAM entries T, T_c^l, X_{mc}^t, $\tau0^t$ at step **242**

Output: $\hat{W}_o^t$ for $t\in T_c^l$

For t from (t_c^l+1) to $(t_c^{l+1}-1)$ with step size +1, do at step **244**

$\Psi^t=X_{mc}^t\backslash X_{\tau0}^t$ and $\boldsymbol{\Upsilon}^t=Y^t-H(:,\tau0^t)X_{\tau0}^t$ where operator : is the colon operator in MATLAB language and it means all rows, and $H(:,\tau0^t)$ denotes a sub-matrix of H constructing from all rows of H with columns identified by the set $\tau0^t$.

$H_o^t=\boldsymbol{\Upsilon}^t\Psi^{t^T}(\Psi^t\Psi^{t^T})^\dagger$ and estimate Ψ^{*t} in the same manner as $\hat{W}_{O\Upsilon}^t=(H_o^t)^\dagger\boldsymbol{\Upsilon}^t$.

$\hat{W}_o^t=\Psi^{*t}\cup X_{\tau0}^t$

End for

Here, to make the best use of the most-current X_{mc}^t, first, optimal-coherent inference module **24** cancels out the direct per-flow statistics **52**, which have been accurately measured, and provide the required $\boldsymbol{\Upsilon}^t$ and Ψ^t for optimal-coherent inference module **24**. Then, the optimal observation matrix **58** of SNMP link-loads H_o^t is computed. Finally, the minimum-norm solution Ψ^{*t} is estimated and traffic matrix **52** or $\hat{W}_O^t$ at time t is constructed.

In all algorithms, the matrix and vector manipulations are match with the syntax of the commands of MATLAB language.

FIG. **8** and FIG. **9** represent the flowchart of algorithms in l^{th} coherence time T_c^l from (T_c^l+1) to $(T_c^{l+1}-1)$, that is, $T_c^l=\{t_c^l+1, \ldots, t_c^{l+1}-1\}$.

FIG. **10** and FIG. **11** show the effectiveness of embodiments of the disclosed invention with real traffic traces from Geant network considered. FIG. **10** shows the performance of a non-adaptive case which is a special case of the adaptive case with only one learning phase in the beginning. FIG. **11** shows the performance of an adaptive case where there are multiple learning phases at the important time of the detection of significant changes.

The framework below provides different metrics for evaluating the performance of the network controller **10**.

$$NMSE = \frac{1}{T_0}\sum_{t=1}^{T_0} NMSE(t) \text{ where } NMSE(t) := \frac{\|X^t - \hat{W}_o^t\|_2}{\|X^t\|_2}$$

NMSE denotes normalized mean squared error and measures the accuracy of flow size estimation technique. Accordingly, $NMSE_{mn}$ denotes the error using the regular minimum norm estimation method where $X_{mn}^t = H^\dagger Y^t = (H^T(HH^T)^{-1})Y^t$, and $NMSE_O$ denotes the estimation error using the optimal-coherent inference module **24**. The $NMSE_{mn}^a$, $NMSE_O^a$ denote the NMSE in the adaptive case, and $NMSE_{mn}^{na}$ and $NMSE_O^{na}$ denote the NMSE in non-adaptive case.

As used in this application, the term "a" or "an" means "at least one" or "one or more."

As used in this application, the term "about" or "approximately" refers to a range of values within plus or minus 10% of the specified number.

As used in this application, the term "substantially" means that the actual value is within about 10% of the actual desired value, particularly within about 5% of the actual desired value and especially within about 1% of the actual desired value of any variable, element or limit set forth herein.

All references throughout this application, for example patent documents including issued or granted patents or equivalents, patent application publications, and non-patent literature documents or other source material, are hereby incorporated by reference herein in their entireties, as though individually incorporated by reference, to the extent each reference is at least partially not inconsistent with the disclosure in the present application (for example, a reference that is partially inconsistent is incorporated by reference except for the partially inconsistent portion of the reference).

Any element in a claim that does not explicitly state "means for" performing a specified function, or "step for" performing a specified function, is not to be interpreted as a "means" or "step" clause as specified in 35 U.S.C. §112, ¶6. In particular, any use of "step of" in the claims is not intended to invoke the provision of 35 U.S.C. §112, ¶6.

While the foregoing written description of the invention enables one of ordinary skill to make and use what is considered presently to be the best mode thereof, those of ordinary skill will understand and appreciate the existence of variations, combinations, and equivalents of the specific embodiment, method, and examples herein. The invention should therefore not be limited by the above described embodiment, method, and examples, but by all embodiments and methods within the scope and spirit of the invention as claimed.

What is claimed is:

1. A method for solving an under-determined inverse problem or network inference/tomography problem in per-flow size, delay, loss and throughput inference in a computer network, through a system; given an original observation matrix, the method comprising:

(a) establishing the computer network having a plurality of nodes wherein the per-flow size, the delay, the loss and the throughput inferences are unknown; wherein an original routing matrix determines a contribution of each flow size on each link-load;

(b) performing a learning phase by:

(1) measuring a set of quantities which are combinations of unknowns creating a first set of measurements;

(2) measuring at least some of a set of unknowns creating a second set of measurements;

(3) using functions of the first set of measurements and functions of the second set of measurements, computing an optimal observation matrix or pseudo-optimal observation matrix;

(c) performing a computer controlled adaptive measurement and inference phase by:

(1) measuring the set of quantities which are the combinations of the unknowns creating measurement quantities;

(2) identifying a partial set of most informative unknown network flows that must be re-measured and updated;

(3) re-measuring a partial set of unknowns;

(4) re-computing and updating the optimal observation matrix to form an updated optimal observation matrix;

(5) estimating the set of unknowns using the measurement quantities, and a function of one of the set consisting of: the optimal observation matrix, the original observation matrix, or both;

wherein, the under-determined inverse problem is a traffic matrix estimation problem and a set of unknown network flows are estimated by:

a) measuring SNMP link loads as measured quantities which are linear combinations of a size of unknown network flows;

b) measuring a set of unknown network flow sizes using counters associated with one of the set consisting of: flow-table entries, TCAM entries or both;

c) computing the optimal observation matrix as a replacement of the original routing matrix; and

d) inferring a set of unknown flows using specific estimation techniques and using SNMP link load measurements, and the function of one of the set consisting of the optimal observation matrix, the original routing matrix, or both.

2. The method of claim **1**, further comprising: (d) determining a starting time of the learning phase and a duration of the learning phase by detecting sequential significant changes in measurements during the computer controlled adaptive measurement and inference phase.

3. The method of claim **1**, further comprising: (f) determining the set of most informative unknown network flows using at least one learning algorithm; wherein the at least one learning algorithm includes a multi-bandit algorithm.

4. The method of claim 1, further comprising generating the optimal observation matrix using one of the set consisting of:

(a) a regularized optimization technique including a least square estimation technique with constraints; and

(b) the regularized optimization technique including the least square estimation technique without the constraints.

5. The method of claim 1, further comprising generating the optimal observation matrix using one of the set consisting of:

(a) a non-regularized optimization technique including a least square estimation technique with constraints; and

(b) the non-regularized optimization technique including the least square estimation technique without the constraints.

6. The method of claim 1, generating an estimation of unknowns using a specified estimation technique using one of the set consisting of:

(a) a regularized optimization technique including a least square estimation technique with constraints; and

(b) the regularized optimization technique including the least square estimation technique without the constraints.

7. The method of claim 1, generating an estimation of unknowns using a specified estimation technique using one of the set consisting of:

(a) a non-regularized optimization technique including a least square estimation technique with constraints; and

(b) the non-regularized optimization technique including the least square estimation technique without the constraints.

8. The method of claim 1, wherein, measurements of the set of unknowns, in both learning phase and computer controlled adaptive measurement and inference phases, are approximated using: 1) mathematical, statistical, heuristic and experimental models, 2) auxiliary information, and 3) information from network topology, distance between nodes, the size of queues, buffers in nodes, and any combination of these.

9. The method of claim 1, wherein, a function of the optimal observation matrix is used for routing a set of network flows.

10. The method of claim 1, wherein, in the learning phase, several vectors of measurement quantities and unknown attributes of interest are available, and the optimal observation matrix or pseudo-optimal observation matrix is calculated using a least mean squares algorithm; wherein, in the computer controlled adaptive measurement and inference phase, a set of new measurements are provided using a function of the optimal observation matrix; wherein the unknowns of interest is estimated using a specific estimation technique.

11. The method of claim 1,

wherein, the under-determined inverse problem is constructed by converting one of the set consisting of: a square inverse problem, over-determined inverse problem or both to an under-determined inverse problem,

wherein, the optimal observation matrix is computed and used for the estimation of unknowns of interest in one of the set consisting of: a square inverse problem, an over-determined inverse problem or both.

12. The method of claim 1, wherein, the under-determined inverse problem is a system identification in the under-determined multi-input and multi-output systems, and a realization of parameters of a model of the system are estimated by computing an optimal observation matrix.

13. The method of claim 12, wherein, the system is an under-determined multi-input and multi-output communication system wherein:

a) in the learning phase, for all possible vectors of communication symbols, the optimal observation matrix is computed as follows:

1) transmitting a vector of known symbols at multiple transmitters;

2) measuring the vector of known symbols at multiple receivers; creating vectors of measurements;

3) computing the optimal observation matrix for each vector of known symbols and a corresponding vector of measurements, creating a plurality of optimal observation matrixes;

4) saving the vector of known symbols and the plurality of optimal observation matrixes at a receiver device;

b) in the computer controlled adaptive measurement and inference phase:

1) transmitting an unknown vector of symbols from multiple transmitters;

2) creating a measurement vector from multiple measurements at multiple receivers;

3) estimating vectors of symbols using the measurement vector and all optimal observation matrices creating estimated vectors of symbols;

4) computing a distance between the estimated vectors of symbols, and the vectors of known symbols;

5) decoding a minimum distance vector of known symbols as an ultimate estimate of the unknown vector of symbols.

14. A system useful for displaying a solution to an under-determined inverse problem in per-flow size, delay, loss and throughput inference in a computer network; the system comprising:

(a) the computer network having a plurality of nodes where the per-flow size, the delay, the loss and throughput estimates inference are unknown;

(b) a computer store configured to store data for at least two or more nodes;

(c) a display configured to display the per-flow size, the delay, the loss and throughput estimates in the computer network;

(d) a computer controller connected to each of the plurality of nodes, wherein the computer controller is coupled to the computer store and programmed to:

(1) perform a learning phase by:

(i) measuring a set of quantities which are combinations of unknowns creating a first set of measurements;

(ii) measuring at least some of a set of unknowns creating a second set of measurements;

(iii) using functions of the first set of measurements and functions of the second set of measurements, and computing an optimal or pseudo-optimal observation matrix;

(2) perform an adaptive measurement and inference phase by:

(i) measuring the set of quantities which are the combinations of the unknowns creating measurement quantities;

(ii) identifying a partial set of most informative unknown network flows that must be re-measured and updated;

(iii) re-measuring the partial set of most informative unknown network flows;

(iv) re-computing and updating an optimal observation matrix to form an updated optimal observation matrix;

(v) estimating the set of unknowns using the measurement quantities, and a function of one of the set consisting of: the optimal observation matrix, an original observation matrix, or both; and

(3) display the unknowns;

wherein, the under-determined inverse problem is a traffic matrix estimation problem and a set of unknown network flows are estimated by:

a) measuring SNMP link loads as measured quantities which are linear combinations of a size of unknown network flows;

b) measuring a set of unknown network flow sizes using counters associated with one of the set consisting of: flow-table entries, TCAM entries or both;

c) computing the optimal observation matrix as a replacement of the original routing matrix; and

d) inferring a set of unknown flows using specific estimation techniques and using SNMP link load measurements, and the function of one of the set consisting of the optimal observation matrix, the original routing matrix, or both.

15. A computer-readable memory adapted for use by a computer network in determining a solution to an under-determined inverse problem in per-flow size, delay, and throughput inference, the computer-readable memory used to direct a computer of the network to perform the steps of:

(a) the computer network having a plurality of nodes where the per-flow size, the delay, the loss and throughput estimates inference are unknown;

(b) a computer store configured to store data for at least two or more nodes;

(c) a display configured to display the per-flow size, the delay, the loss and throughput estimates in the computer network;

(d) a computer controller connected to each of the plurality of nodes, wherein the computer controller is coupled to the computer store and programmed to:

(1) perform a learning phase by:

(i) measure a set of quantities which are combinations of unknowns creating a first set of measurements;

(ii) measure at least some of a set of unknowns creating a second set of measurements;

(iii) use functions of the first set of measurements and functions of the second set of measurements, and computing an optimal or pseudo-optimal observation matrix;

(2) perform an adaptive measurement and inference phase by:

(i) measuring the set of quantities which are the combinations of the unknowns creating measurement quantities;

(ii) identifying a partial set of most informative unknown network flows that must be re-measured and updated;

(iii) re-measuring a partial set of most informative unknowns;

(iv) re-computing and updating an optimal observation matrix to form an updated optimal observation matrix;

(v) estimating the set of unknowns using the measurement quantities, and a function of one of the set consisting of: the optimal observation matrix, an original observation matrix, or both; and

(3) display the unknowns;

wherein, the under-determined inverse problem is a traffic matrix estimation problem and a set of unknown network flows are estimated by:

a) measuring SNMP link loads as measured quantities which are linear combinations of a size of unknown network flows;

b) measuring a set of unknown network flow sizes using counters associated with one of the set consisting of: flow-table entries, TCAM entries or both;

c) computing the optimal observation matrix as a replacement of the original routing matrix; and

d) inferring a set of unknown flows using specific estimation techniques and using SNMP link load measurements, and the function of one of the set consisting of the optimal observation matrix, the original routing matrix, or both.

* * * * *